Complex geometry and operator theory

Joseph A. Ball, Department of Mathematics, Virginia Tech: *Realization and interpolation theory for the Herglotz-Agler class over the poly-right-halfplane*

Abstract. The Schur-Agler class of functions is defined as the class of holomorphic functions S on the polydisk $\mathbb{D}^d$ for which $S(T_1, \ldots, T_d)$ has norm at most 1 whenever $T_1, \ldots, T_d$ is a commutative tuple of strict contraction operators on a Hilbert space $\mathcal{H}$. The Herglotz-Agler class is the class of holomorphic functions H on the d-variable poly-right-halfplane for which $H(X_1, \ldots, X_d)$ has positive real part whenever $X_1, \ldots, X_d$ is a commutative family of operators with each having strictly positive real part. While the Herglotz-Agler class is just a linear-fractional transform of the Schur-Agler class and the realization theory for the Schur-Agler class (i.e., realization of S as the transfer function of a multidimensional conservative input/state/output linear system) is well understood, the realization theory for the Herglotz-Agler class is considerably more subtle, especially in the several-variable case. We discuss several approaches to the realization theory for the Herglotz-Agler class with special attention to the rational case, and also indicate connections with a homogeneous subclass (the so-called Bessmertnyi class) of the Herglotz-Agler class.

Sergey Belyi, Department of Mathematics, Troy University: *On sectorial classes of inverse Stieltjes functions*

Abstract. We introduce sectorial classes of inverse Stieltjes functions acting on finite-dimensional Hilbert space as well as scalar classes of inverse Stieltjes functions characterized by their limit values. It is shown that a function from these classes can be realized as the impedance function of an L-system whose associated operator $\tilde{\mathbb{A}}$ is sectorial. Moreover, it is established that the knowledge of the limit values of the scalar impedance function allows us to find an angle of sectoriality of operator $\tilde{\mathbb{A}}$ as well as the exact angle of sectoriality of the accretive main operator T of such a system. The corresponding new formulas connecting the limit values of the impedance function and the angle of sectoriality of $\tilde{\mathbb{A}}$ are provided. These results are illustrated by examples of the realizing L-systems based upon the Schrödinger operator on half-line.

The talk is based on a recent joint work with E. Tsekanovskiĭ (see [1]).

[1] S.V. Belyi, E.R. Tsekanovskiis. Sectorial classes of inverse Stieltjes functions and L-systems. *Methods of Functional Analysis and Topology* (2012) (to appear).

Tirthankar Bhattacharyya, Department of Mathematics, Indian Institute of Science: *The role of the fundamental operator in functional model and complete unitary invariance of a pure* Γ*-contraction*

Abstract. A pair of commuting operators (S, P) defined on a Hilbert space $\mathcal{H}$ for which the closed symmetrized bidisc

$$\Gamma = \{(z_1 + z_2, z_1 z_2) : |z_1| \leq 1, \, |z_2| \leq 1\} \subseteq \mathbb{C}^2,$$

is a spectral set is called a Γ-contraction in the literature. A Γ-contraction (S, P) is said to be pure if P is a pure contraction, i.e, $P^{*n} \to 0$ strongly as $n \to \infty$. Here we construct a functional model and produce a complete unitary invariant for a pure Γ-contraction. The key ingredient in these constructions is an operator, which is the unique solution of the operator equation

$$S - S^* P = D_P X D_P, \text{ where } X \in \mathcal{B}(\mathcal{D}_P),$$

and is called the fundamental operator of the Γ-contraction (S, P). This talk is based on joint work with Sourav Pal.

Greg Knese, Department of Mathematics, University of Alabama: *Operator theory and Pick interpolation on distinguished varieties*

Abstract. Distinguished varieties are algebraic curves that exit the bidisk through the distinguished boundary. They offer a natural setting to generalize familiar one variable operator related function theory. We will survey joint work with Agler and McCarthy on pairs of isometries satisfying a polynomial and the connection to distingiushed varieties. We will also survey joint work with Jury and McCullough, as well as work by others, on Pick interpolation in this setting.

Zinaida A. Lykova, School of Mathematics, Newcastle University: *Extremal holomorphic maps and the symmetrised bidisc*

Abstract. We introduce the class of n-extremal holomorphic maps, a class that generalises both finite Blaschke products and complex geodesics, and apply the notion to the finite interpolation problem for analytic functions from the open unit disc into the symmetrised bidisc Γ. We show that a well-known necessary condition for the solvability of such an interpolation problem is not sufficient whenever the number of interpolation nodes is 3 or greater. We introduce a sequence $\mathcal{C}_\nu, \nu \geq 0$, of necessary conditions for solvability, prove that they are of strictly increasing strength and show that $\mathcal{C}_{n-3}$ is insufficient for the solvability of an n-point problem for $n \geq 3$.

We introduce a classification of rational Γ-inner functions, that is, analytic functions from the disc into Γ whose radial limits at almost all points on the unit circle lie in the distinguished boundary of Γ. The classes are related to n-extremality and the conditions $\mathcal{C}_\nu$; we present numerous strict inclusions between the classes. The talk is based on a joint work with Jim Agler and N. J. Young.

Gadadhar Misra, Department of Mathematics, Indian Institute of Science:*Homogeneous operators in the Cowen-Douglas class of the ball*

Abstract. We describe, via the jet construction, a new class of commuting tuple $\mathbf{T}$ of operators in the Cowen-Douglas class which are homogeneous with respect to the automorphism group of the ball, that is, $\varphi \cdot \mathbf{T}$ is unitrily equivalent to $\mathbf{T}$ for all φ in the automorphism group of the ball, $\varphi \cdot \mathbf{T}$ is the natural action of φ on $\mathbf{T}$ via the holomorphic functional calculus.

Sourav Pal, Stat-Math Unit, Indian Statistical Institute:
On Gamma contractions

Abstract. A pair of bounded operators (S, P) for which the closed symmetrized bidisc

$$\Gamma = \{(z_1 + z_2, z_1 z_2) : |z_1| \leq 1, |z_2| \leq 1\} \subseteq \mathbb{C}^2$$

is a spectral set, is called a Γ-contraction in the literature. For a contraction P and a bounded commutant S of P, let us consider the operator equation

$$S - S^* P = (I - P^* P)^{\frac{1}{2}} X (I - P^* P)^{\frac{1}{2}},$$

where X is a bounded operator on $\overline{\mathrm{Ran}}(I - P^* P)^{\frac{1}{2}}$ with numerical radius of X being not greater than 1. We show that the existence and uniqueness of

solution to the operator equation for a Γ-contraction (S, P) leads us to an explicit construction of a minimal Γ-isometric dilation of the Γ-contraction (S, P). Hence it follows that the existence and uniqueness of solution to the above operator equation is a necessary and sufficient condition for the set Γ to be a spectral set for a pair of bounded operators.

Orr Shalit, Department of Mathematics, Ben-Gurion University of the Negev: *Operator algebraic complex geometry*

Abstract. Let M be an algebra of holomorphic functions on the unit ball in complex n-space. Let V be an analytic variety in the ball. Restricting the algebra M to V, one gets an algebra of functions on V, call it $M(V)$.

Problem: how does the geometry of V determine the structure of $M(V)$?

In general, it depends what one means by "geometry" and what one means by "structure". We are interested in the operator algebraic, the Banach algebraic and the sheer algebraic structures of $M(V)$, and these different types of structure correspond to different interpretations of geometry. In the special case where M is the algebra of multipliers on Drury-Arveson space, and V is a homogeneous algebraic variety, we can say that the geometry of V completely determines the structure of $M(V)$, and vice-versa. When V is not homogeneous the situation is far more delicate, as I will explain. Our investigations on this matter have also led us to results in complex geometry of independent interest.

The talk is based on joint work with Ken Davidson and Chris Ramsey.

Nicholas Young, Department of pure mathematics, Leeds university and School of Mathematics, Newcastle University:
Domains associated with mu-synthesis and magic functions

Abstract. The problem of mu-synthesis arises in H^∞ control. It is a species of interpolation problem which generalizes classical problems, such as those of Nevanlinna-Pick and Carathéodory-Fejér, but is much harder. Attempts to solve this problem have led to the study of some previously unfamiliar domains in $\mathbb{C}^n$. Two of these, the symmetrised polydisc and the tetrablock, have now been extensively studied by both operator-theorists and specialists in several complex variables, but there are others too. I shall describe some of the background to these domains, give some examples, describe the notion of magic functions of a domain and explain its connection

with complex geometry, and particularly with the problem of whether the invariant distances on a given domain coincide.