UM 101 : Analysis and Linear Algebra I

August - December 2012

Indian Institute of Science

Exercises 4

31 August, 2012

1. Give an example of a function f such that f is continuous nowhere, but $|f|$ is continuous everywhere.

2. Suppose that f and g are functions such that g is continuous at $0, g(0) = 0$, and $|f(x)| \leq |g(x)|$ for all x. Prove that f is continuous at 0.

3. For each real number a, find a function which is continuous at a, but not continuous elsewhere.

4. Find a function f which is discontinuous at $1, \frac{1}{2}, \frac{1}{3}, \ldots$, but continuous at all other points.

5. Find a function f which is discontinuous at $1, \frac{1}{2}, \frac{1}{3}, \ldots$, and at 0 , but continuous at all other points.

6. Suppose that a function f satisfies $f(x+y) = f(x)+f(y)$ for all x, y, and that f is continuous at 0. Show that f is continuous everywhere.

7. Suppose that f is continuous at a and $f(a) = 0$. Show that if $\alpha \neq 0$, then $f + \alpha$ is nonzero in some open interval containing a.

8. Prove that if f is continuous at a, then so is $|f|$. Is the converse of this statement true?

9. Prove that if f and g are continuous, then so are $\max(f, g)$ and $\min(f, g)$.

10. Prove that every continuous f can be written $f = g - h$, where g and h are nonnegative and continuous.