# UM 101 : Analysis and Linear Algebra I

August - December 2012

Indian Institute of Science

Exercises 11

26 October, 2012

1. Differentiate each of the following functions.
   (a) $f(x) = e^{(\int_0^x e^{-t^2}\, dt)}$.
   (b) $f(x) = (\ln x)^{\ln x}$.
   (c) $f(x) = x^x$.
2. Find $\lim_{x\to\infty} x \sin \frac{1}{x}$.
3. Find
$$\int_a^b \frac{f'(t)}{f(t)}\, dt$$
for $f > 0$ on $[a, b]$.
4. Show that
$$F(x) = \int_2^x \frac{1}{\ln t}\, dt$$
is not bounded on $[2, \infty)$.
5. Suppose that $f''$ is continuous and that
$$\int_0^\pi [f(x) + f''(x)] \sin x\, dx = 0.$$
Given that $f(\pi) = 1$, compute $f(0)$.
6. Prove that
$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx.$$
7. Find the Taylor polynomial of the function $f(x) = e^{\sin x}$ of degree 3 at 0.
8. Prove that the Taylor polynomial of $f(x) = \sin(x^2)$ of degree $4n + 2$ at 0 is
$$x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \ldots + (-1)^n \frac{x^{4n+2}}{(2n+1)!}.$$
9. Prove that if $x \leq 0$, then
$$\left| \int_0^x \frac{e^t}{n!}(x - t)^n\, dt \right| \leq \frac{|x|^{n+1}}{(n+1)!}.$$
10. Prove that if $-1 < x \leq 0$, then
$$\left| \int_0^x \frac{t^n}{1+t}\, dt \right| \leq \frac{|x|^{n+1}}{(1+x)(n+1)}.$$