

UM 101 : Analysis and Linear Algebra I
August - December 2012
Indian Institute of Science
Exercises 10

19 October, 2012

1. Show that the value of

$$\int_{-\cos x}^{\sin x} \frac{1}{\sqrt{1-t^2}} dt$$

does not depend on x .

2. Find $(f^{-1})'(0)$ if

$$f(x) = \int_1^x \cos(\cos t) dt.$$

3. Find all continuous functions f satisfying

$$\int_0^x f(t) dt = (f(x))^2 + C.$$

4. Prove that if f is continuous, then

$$\int_0^x f(u)(x-u) du = \int_0^x \left(\int_0^u f(t) dt \right) du$$

5. Find a function f such that $f'''(x) = \frac{1}{\sqrt{1+\sin^2 x}}$.

6. Find a function g such that

$$\int_0^{x^2} tg(t) dt = x + x^2.$$

7. Find all $x > 0$ for which $\int_0^x [t]^2 dt = 2(x-1)$.

8. Show that $\int_0^x (t+|t|)^2 dt = 2x^2(x+|x|)/3$ for all real x .

9. A function f is continuous everywhere and satisfies

$$\int_0^x f(t) dt = -1/2 + x^2 + x \sin(2x) + 1/2 \cos(2x)$$

for all x . Find $f(\pi/4)$ and $f'(\pi/4)$.

10. Given a function g , continuous everywhere, such that $g(1) = 5$ and $\int_0^1 g(t) dt = 2$. Let

$$f(x) = (1/2) \int_0^x (x-t)^2 g(t) dt.$$

Prove that

$$f'(x) = x \int_0^x g(t) dt - \int_0^x tg(t) dt.$$

Then find the value of $f''(1)$.