IWOTA 2013

Plenary speakers, their Titles and Abstracts

M. Jean-Pierre Antoine, Louvain, Belgium.

Title: ***Coherent states and wavelets, a contemporary panorama.***

Abstract: In a first part, we review the general theory of coherent states (CS). Starting from the canonical CS introduced by Schrödinger in 1926 and rediscovered by Glauber, Klauder, Sudarshan in the 1960s, we proceed to the derivation of general CS from square integrable group representations and some of their generalizations : Perelomov CS, general square integrable covariant CS, nonlinear CS, Gazeau-Klauder CS, with a hint to their application in quantization.

Next, we turn to signal processing and note that the two most familiar tools, namely, the Gabor transform and the wavelet transform, are special cases of CS, associated to the Weyl-Heisenberg group (which yields the canonical CS) and the affine group of the line, respectively. Then we review the properties of the wavelet transform, both in its continuous and its discrete versions, in one or two dimensions, emphasizing mostly the mathematical properties. We also consider its extension to higher dimensions, to more general manifolds (sphere, hyperboloid,...) and to the space-time context, for the analysis of moving objects.

Rajendra Bhatia, Delhi, India.

Title: ***Inertias of some special matrices.***

Abstract: Let $p_1, \ldots, p_n$ be positive real numbers, and for each real number r consider the $n \times n$ matrix $P_r = [(p_i + p_j)^r]$. The matrix P_{-1} is called the Cauchy matrix. The special case $p_i = i$ of this is the famous Hilbert matrix.

It is well-known that for each $r < 0$, the matrix P_r is positive definite. Much more intriguing is the behaviour of the spectrum of P_r when $r > 0$. That will be the main theme of this talk.

A related problem about Loewner matrices will also be discussed.

Isabelle Chalendar, Lyon, France.

Title: ***Inner functions in Operator Theory.***

Abstract: An inner function is a bounded analytic function on the unit disk whose radial limits have modulus one at almost every point of the unit circle. Inner functions arose originally in an operator-theoretic context, via Beurling's characterization of the invariant subspaces of the unilateral shift. Since then, they have played important roles in a number of operatorial contexts. I shall present some of them (in collaboration with P. Gorkin and J. Partington), including the theory of model spaces and restricted shifts, certain interesting questions about interpolation by analytic functions, as well as the theory of composition operators on Hardy spaces.

Raúl E. Curto, Iowa, USA.

Title: ***Berger measures for transformations of subnormal weighted shifts.***

Abstract: A subnormal weighted shift may be transformed to another shift in various ways, such as taking the p-th power of each weight or forming the Aluthge transform. In joint work with George Exner (Bucknell University, USA), we determine in a number of cases whether the resulting shift is subnormal, and, if it is, we find a concrete representation of the associated Berger measure. We do this directly for finitely atomic measures, and using both Laplace transform and Fourier transform methods for more complicated measures. Alternatively, the problem may be viewed in purely measure theoretic terms as the attempt to solve moment matching equations such as $(\int t^n \, d\mu(t))^2 = \int t^n \, d\nu(t)$ $(n = 0, 1, \ldots)$ for one measure given the other.

Birgit Jacob, Wuppertal, Germany.

Title: ***Linear port-Hamiltonian systems on Infinite-dimensional spaces.***

Abstract: Modeling of dynamical systems with a spatial component leads to lumped parameter systems, when the spatial component may be denied, and to distributed parameter systems otherwise. The mathematical model of distributed parameter systems will be a partial differential equation. Examples of dynamical sytems with a spatial component are, among others, temperature distribution of metal slabs or plates, and the vibration of aircraft wings. In this talk we will study distributed parameter portHamiltonian systems. This class contains the above mentioned examples. The norm of such a system is given by the energy (Hamiltonian) of the system. This fact enables us to show relatively easy the existence and stability of solutions. Further, it is possible to determine which boundary variables are suitable as inputs and outputs, and how the system can be stabilized via the boundary.

James E. Jamison, Memphis, USA.

Title: ***Hermitian operators on vector valued function spaces.***

Abstract: In this talk I will present some recent results on Hermitian operators (bounded and unbounded) on various vector valued functions spaces. In particular, I will focus on the spaces $H^p_{\mathcal{H}}(D^n)$ (the vector valued Hardy space of the n-disk) and $B_0(\triangle, E)$, the vector valued little Bloch space . In the later case, the surjective isometries will first be characterized. Some related questions will be discussed.This is joint work with Fernanda Botelho.

Gilles Pisier, College Station, USA.

Title: ***Quantum Expanders and Growth of Operator Spaces.***

Abstract: We show that there are well separated families of quantum expanders with asymptotically the maximal cardinality allowed by a known upper bound. This allows us to provide sharp estimates for the growth of the multiplicity of M_N-spaces needed

to represent (up to a constant $C > 1$) the M_N-version of the n-dimensional operator Hilbert space OH_n as a direct sum of copies of M_N. We show that, when C is close to 1, this multiplicity grows as $\exp \beta n N^2$ for some constant $\beta > 0$. The methods have a "geometric" flavor analogous to familiar Euclidean pictures. The main idea is to relate quantum expanders with "smooth" points on the matricial analogue of the unit sphere. This generalizes to operator spaces a classical geometric result on n-dimensional Hilbert space (corresponding to $N = 1$). We will also discuss another notion of growth more adapted to measure the lack of "exactness" of operator spaces. Lastly, we give estimates of the metric entropy of the space of n-dimensional operator spaces equipped with the matricial analogue of the Banach-Mazur distance.

Dinesh Singh, Delhi, India.

Title: ***Operators on Function Spaces and Applications of the H^1 - BMOA Duality.***

Abstract: This talk is based on joint work and it shall expose a number of new results that relate to properties of functions and operators on the Hardy spaces where a critical tool is the H^1 - BMOA duality.

Kalyan B. Sinha, Bangalore, India.

Title: ***An Approximation Theorem and Two-variable Trace Formula.***

Abstract: For a commuting n-tuple of bounded self adjoint operators, a commuting finite-dimensional n-tuple of self-adjoint operators is constructed such that it approximates the given tuple, in an appropriate sense, in Schatten - $\mathcal{B}_n$ - norm . This result is then used to prove an extension of Krein-Stokes like trace formula for two variables.

V. S. Sunder, Chennai, India.

Title: ***Minimax theorems in Non-commutative Probability Spaces .***

Abstract: This talk will be on joint work with Madhushree Basu on extending various classical minimax characterisations - due to Courant-Fisher-Weyl, Ky Fan, and Wielandt - of (sums of) eigenvalues of selfadjoint matrices, to the context of non-commutative probability spaces.

Harald Upmeier, Marburg, Germany.

Title: ***Hilbert spaces of cohomology and Radon transform .***

Abstract: Hilbert spaces of holomorphic functions, such as Hardy spaces and Bergman spaces, play a fundamental role in operator theory and complex analysis in several variables. These Hilbert spaces belong to domains which are pseudo-convex. In more general situations, occurring in the decomposition of the L2-space over the Shilov boundary, one encounters non-convex domains which do not carry holomorphic functions but d-bar cohomology classes. In the talk it is shown that integral transformations generalizing the classical Radon transform can be used to analyze Dolbeault cohomology spaces and their boundary values. Basic examples are non-convex tube domains and domains in Grassmann manifolds.

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Semi-Plenary speakers, their Titles and Abstracts

S. Twareque Ali, Montreal, Canada.

Title: ***Some families of complex Hermite polynomials and their applications to physics.***

Abstract: We discuss two families of Hermite polynomials in a complex variable which are orthogonal, and more generally, biorthogonal with respect to certain measures on the complex plane. They span interesting Hilbert spaces of holomorphic functions, leading to non-conventional quantizations of the complex plane. Additionally, they generate the much studied squeezed states of quantum optics.

Daniel Alpay, Be'er-Sheva, Israel.

Title: ***Non commutative stochastic distributions, free processes with stationary increments and stochastic integration.***

Abstract: A recent approach to linear stochastic systems (where randomness is also included in the transfer function) uses Hida white noise space theory. In this approach, the complex numbers are replaced by the Kondratiev algebra of stochastic distributions. Hida white noise space is the symmetric Fock space associated to the Lebesgue space, and form, with the Kondratiev spaces of stochastic test functions and stochastic distributions, a Gelfand triple. This setting allows to develop the stochastic calculus for a wide family of stationary increments stochastic processes and a new theory of linear stochastic systems. In the talk we report on the free version of the above approach, where now the white noise space is replaced by the full Fock space and one considers free processes with stationary increments. The commutative and non commutative Kondratiev spaces are examples of a new family of topological algebras introduced by the author and Guy Salomon. These algebras are inductive limits of Banach spaces and carry inequalities which are counterparts of the inequality for the norm in a Banach algebra. Their definition and properties will be briefly outlined at the end of the talk. The talk is based on joint works with H. Attia, P. Jorgensen, D. Levanony and G. Salomon.

Wolfgang Arendt, Ulm, Germany.

Title: ***Regularity of Semigroups : Asymptotic Behaviour at*** 0 ***and Multipliers.***

Abstract: In this talk we consider semigroups on Banach spaces. The main emphasis are questions of regularity. At first we talk about holomorphy which has a very convenient characterization by a simple estimate of the resolvent of the generator, but also by the asymptotic behavior of the semigroup at 0. Then we explain a very actual subject, namely maximal regularity. On Hilbert spaces it is nothing more than holomorphy, but on other Banach spaces it is. This is a deep result by Kalton-Lancien, which has a new more concrete proof given by Fackler recently. We will explain how maximal regularity can be characterized on (UMD-) Banach spaces (Theorem of L. Weis) by a resolvent estimate involving R-boundedness but also a new characterization by the asymptotic behavior of the semigroup at 0. One possible approach to maximal regularity is based on the periodic operator-valued Marcinkiewicz multiplier theorem which we present in a new abstract form. An operator theoretical application is the description of generators of periodic groups on UMD spaces.

Joseph A. Ball, Blacksburg, Virginia, USA.

Title: ***Transfer-function realization and zero/pole structure for multivariable rational matrix functions: the direct analysis.***

Abstract: An elementary fact in complex analysis is that a nonzero rational function of a complex variable is completely determined (up to a nonzero constant factor) by its poles and zeros if one includes multiplicity information. Extension of this idea to the case of a matrix-valued rational function having determinant not vanishing identically was worked out by Israel Gohberg and collaborators in the 1980s (see [1]). In addition to location of poles and zeros as well as multiplicities, one must specify pole-directional, zero-directional, and pole/zero coupling information; then one can recover the rational matrix function up to an invertible constant factor via an explicit formula in the form of the transfer function of an input/state/output linear system. When specialized to the elementary scalar case, one arrives at the solution in a partial-fraction form rather than a multiplicative form. Here I present ongoing work with Quanlei Fang and Victor Vinnikov on further extensions of these ideas to the multivariable case, namely, to the study of a rational matrix function in several complex variables with determinant not vanishing identically. The analysis requires nontrivial constructs from algebraic geometry (zero and pole varieties, schemes, and sheaves) as well as transfer functions of multidimensional linear systems (specifically, Fornasini-Marchesini realizations). This lecture focuses on the direct analysis; the companion lecture of Victor Vinnikov at this IWOTA will pursue the converse analysis on how zero/pole information leads to the actual construction of transfer-function realizations for a given nondegenerate multivariable rational matrix function.

[1] J.A. Ball, I. Gohberg, and L. Rodman, *Interpolation of Rational Matrix Functions*, Oper. Theory Adv. Appl. **45**, Biirkhäuser-Verlag, 1990.

Paul Binding, Calgary, Canada.

Title: ***Some two parameter eigenvalue problems.***

Abstract: We review some ideas involving m self-adjoint pencils coupled by n eigenparameters, aiming mainly at the case $m = n = 2$. Starting with connections between some properties of indefinite pencils ($m = n = 1$) and those of special two parameter embeddings ($m = 1, n = 2$), we give some extensions to the case $m = n = 2$ via an embedding with $m = 2, n = 3$. An important technique (due to Hilbert in a special case with $m = n = 2$) leads to PDE if the original pencils correspond to ODE. This technique has since been extended to various abstract tensor product settings and applied to several completeness and expansion questions for eigenvectors and associated vectors.

Here we shall use this technique, in conjunction with eigencurves of two parameter pencils, both in the original and tensor product settings, to shed light on different issues. For example, we give precise relations between the numbers of nonreal eigenpairs and of real ones corresponding to a range of "indices" (giving pairs of eigenfunction oscillation numbers in the ODE case).

Michael A. Dritschel, Newcastle upon Tyne, UK

Title: ***Dilations and constrained algebras.***

Abstract: It is well known that contractive representations of the disk algebra are completely contractive. The Neil algebra A is the subalgebra of the disk algebra consisting of those functions f for which $f'(0) = 0$. There is a complete isometry from the algebra $R(W)$ of rational functions with poles off of the distinguished variety $W = \{(z, w) : z^2 = w^3, |z| < 1\}$ to A. We prove that there are contractive representations of A which are not completely contractive, and furthermore provide a Kaiser and Varopoulos inspired example whereby z and w in W are contractions, yet the resulting representation of $R(W)$ is not contractive. We also present a characterization of those contractive representations which are completely contractive. Finally, we show that for the variety $V = \{(z, w) : z^2 = w^2, |z| < 1\}$, all contractive representations of the algebra $R(V)$ of rational functions with poles off V are completely contractive, and also provide a simplified proof of Agler's analogous result over an annulus.

This is joint work with Michael Jury and Scott McCullough.

Ken Dykema, College Station, USA.

Title: ***Hyperinvariant subspaces and upper triangular decompositions in finite von Neumann algebras.***

Abstract: Building on powerful results of Haagerup and Schultz, we realize an "upper triangular" type decomposition of elements in finite von Neumann algebras. In particular, each such element T can be written as T=N+Q where N is a normal element whose distribution equals the Brown measure of T and where Q is an s.o.t.-quasinilpotent element. This decomposition behaves well with respect to holomorphic functional calculus. (joint work with Fedor Sukochev and Dmitriy Zanin)

Tom ter Elst, Auckland, New Zealand.

Title: ***The Dirichlet-to-Neumann operator via hidden compactness.***

Abstract: Let Δ^D be the Dirichlet Laplacian on a bounded Lipschitz domain Ω with boundary $\Gamma = \partial\Omega$. For all $\lambda \in \mathbb{R} \setminus \sigma(-\Delta^D)$ the Dirichlet-to-Neumann operator is a self-adjoint operator in $L_2(\Gamma)$ with graph

$$D_\lambda = \{(g, h) \in L_2(\Gamma) \times L_2(\Gamma) : \text{there exists a } u \in H^1(\Omega) \text{ such that } -\Delta u = \lambda\, u \text{ weakly in } \Omega,\ \mathrm{Tr}\, u = g \text{ and } \partial_\nu u = h\},$$

where $\partial_\nu u$ is the outer normal derivative. An alternative convenient way to describe this operator is via form methods (which we shall explain in detail in the talk). Define the

form $\mathfrak{a}_\lambda \colon H^1(\Omega) \times H^1(\Omega) \to \mathbb{C}$ by

$$\mathfrak{a}_\lambda(u, v) = \int_\Omega \nabla u \cdot \overline{\nabla v} + \lambda \int_\Omega u\,\overline{v}.$$

Then

$$\begin{aligned} D_\lambda = \{(g, h) \in L_2(\Gamma) \times L_2(\Gamma) : \ & \text{there exists a } u \in H^1(\Omega) \text{ such that} \\ & \operatorname{Tr} u = g \text{ and} \\ & \mathfrak{a}_\lambda(u, v) = \textstyle\int_\Gamma h\,\overline{\operatorname{Tr} v} \text{ for all } v \in H^1(\Omega)\}. \end{aligned}$$

One of the aims of this talk is to give a meaning to D_λ if $\lambda \in \sigma(-\Delta^D)$. In that case D_λ will be a self-adjoint graph (= possibly multi-valued operator). For the graph D_λ the form $\mathfrak{a}_\lambda$ and the trace map $\operatorname{Tr} : H^1(\Omega) \to L_2(\Gamma)$ are needed. If $\lambda \in \sigma(-\Delta^D)$ then the form $\mathfrak{a}_\lambda$ is no longer coercive with respect to the trace map, even if one restricts to a suitable subspace. The key component which allows to take $\lambda \in \sigma(-\Delta^D)$ is that there is a 'hidden compactness' in the sense that the form $\mathfrak{a}_\lambda$ is coercive with respect to a compact map in another Hilbert space, which is in this case the embedding of $H^1(\Omega)$ into $L_2(\Omega)$. This compact map implies a Fredholm alternative and can be used as a substitute for the Lax–Milgram lemma.

This is joint work with W. Arendt, J. Kennedy and M. Sauter.

Debashish Goswami, Kolkata, India.

Title: ***Quantum Isometry Groups.***

Abstract: The quantum isometry group of a given (possibly noncommutative) compact Riemannian manifold is the universal object in the category of compact quantum groups acting 'isometrically' (in a suitable sense) on the underlying C* algebra. The precise formulation including the motivation, existence, tools for computing such objects and a few concrete examples, will be briefly discussed. Time permitting, we also plan to give a sketch of proof of the remarkable recent result that there is no genuine quantum group of isometries acting faithfully on a classical compact connected Riemannian manifold. Based on a series of articles written jointly with J. Bhowmick, B. Das, S.Joardar, A. Skalski and T. Banica.

Fumio Hiai, Tohoku, Japan.

Title: ***Higher order extension of Löwner's theory: Operator k-tone functions.***

Abstract: We present a higher order extension of the theory of Löwner and Kraus on operator/matrix monotone and convex functions. To do so, we introduce the notion of operator/matrix k-tone functions so that when $k = 1$ and $k = 2$, operator k-tone functions are operator monotone and convex functions, respectively. We show differentiability properties of matrix k-tone functions of order n. Such a function must be of class $C^{2n+2k-4}$. In particular, an n-convex function is C^{2n-2}, extending the classical result of Kraus when $n = 2$. Several characterizations of operator k-tone functions on (a, b) are presented in terms of derivatives or divided differences of f. It then turns out that operator k-tone functions have rather simple structure with only additive and

multiplicative polynomial factors beyond operator monotone functions. Integral expressions of operator k-tone functions! on $(-1, 1)$ and on $(0, \infty)$ are given, generalizing the well-known versions for operator monotone functions (when $k = 1$). We also clarify the operator versions of absolutely monotone and completely monotone functions on $(-1, 1)$ and on $(0, \infty)$. Finally, we discuss the behavior of the analytic functional calculus by operator k-tone functions. This talk is based on joint work with U. Franz and É. Ricard.

Il Bong Jung, Taegu, Korea.

Title: ***Unbounded quasinormal operators and related properties.***

Abstract: Various characterizations of unbounded closed densely defined operators commuting with the spectral measures of their moduli are established. In particular, Kaufman's definition of an unbounded quasinormal operator is shown to coincide with that given by Stochel-Szafraniec. Examples demonstrating the sharpness of our results are constructed. An absolute continuity approach to quasinormality which relates the operator in question to the spectral measure of its modulus is developed. Algebraic characterizations of some classes of operators that emerge in this context are found. Various examples and counterexamples illustrating the concepts of this work are constructed by using weighted shifts on directed trees. Generalizations of these results that cover the case of q-quasinormal operators are established. (This is joint work with Z. Jablonski and J. Stochel.)

M. A. Kaashoek, Amsterdam, The Netherlands.

Title: ***The inverse problem for Ellis-Gohberg orthogonal matrix functions.***

Abstract: In a 1992 paper, R.J. Ellis and I. Gohberg introduced a new family of orthogonal functions related to infinite Hankel matrices and with properties that are analogous to those of the classical Szego-Krein polynomials. In particular, they solved the corresponding inverse problem, i.e., the problem of finding necessary and sufficient conditions in order that a given function is an Ellis-Gohberg orthogonal function. The latter includes the problem of reconstructing the corresponding infinite Hankel matrix. In later papers, jointly with D.C. Lay, the matrix-valued version of these Ellis-Gohberg orthogonal functions were studied, but the associate inverse problem remained open except for some special cases. In this talk we will give a systematic analysis of the inverse problem for the matrix-valued case. It turns out that the problem can be reduced to a linear matrix equation with a sp! ecial right hand side. Using this reduction we shall present the solution under the additional requirement that the corresponding (block) Hankel matrix is uniquely determined by the data. In that case the necessary and sufficient conditions are natural generalizations of those appearing in the scalar case. Various examples will be given to illustrate the main results and some open problems will be presented. The talk is based on recent joint work with Freek van Schagen.

Igor Klep, Auckland, New Zealand.

Title: ***Linear Matrix Inequalities and Positive Polynomials.***

Abstract: The talk concerns the classical question of real algebraic geometry: given polynomials p and q, is p positive where q is positive? We focus on free polynomials evaluated at tuples of matrices of all sizes. In case the positivity set $\mathcal{D}_q = \{X : q(X) \succeq 0\}$ underlying q is convex, it is essentially represented as the solution set $\mathcal{D}_L$ of a Linear Matrix Inequality (LMI) $L(x) \succ 0$. LMIs are common in many areas: control systems, combinatorial optimization, statistics, etc.

We present a precise characterization (*Positivstellensatz*) for polynomials p positive on a convex semialgebraic set intersect a variety. Given a generic LMI domain $\mathcal{D}_L$ we determine all "(strong sense) defining polynomials" p for $\mathcal{D}_L$. Such polynomials must have the form

$$p = L\left(\sum_i^{\text{finite}} q_i^* q_i\right) L + \sum_j^{\text{finite}} (r_j L + C_j)^* L (r_j L + C_j),$$

where q_i, r_j are matrices of polynomials, and C_j are real matrices satisfying $C_j L = L C_j$.

This follows from our general result for a given linear pencil L and a finite set I of rows of polynomials. A matrix polynomial p is positive where L is positive and all $\iota \in I$ vanish if and only if

$$\text{(P)} \qquad p = \sum_i^{\text{finite}} p_i^* p_i + \sum_j^{\text{finite}} q_j^* L q_j + \sum_k^{\text{finite}} (r_k^* \iota_k + \iota_k^* r_k),$$

where each p_i, q_j and r_k are matrices of polynomials, and each ι_k is an element of the "L-real radical" of I. In this representation, we can restrict p_i, q_i, ι_k and r_k to be elements of a low-dimensional subspace of matrices of polynomials, and in particular, their degrees depend in a very tame way only on the degree of p and the degrees of the elements of I. Further, this paper gives an efficient algorithm for computing the L-real radical of I.

The theory developed has a number of additional consequences which will be discussed time permitting. For example, it yields algebraic certificates for completely positive maps between nonunital subspaces of matrix algebras.

This is based on joint works with Bill Helton, Scott McCullough, Chris Nelson, and Markus Schweighofer.

Pierre Portal, Canberra, Australia.

Title: ***Holomorphic functional calculus and square functions.***

Abstract: For bounded linear operators acting on Banach spaces, a holomorphic functional calculus can easily be defined through Cauchy's formula, using a contour integral around the spectrum. In applications to PDE, it is important to extend this functional calculus to certain unbounded operators in order to treat differential operators acting on function spaces. The difficulty is that one wants this extension to be bounded, i.e. to define a bounded algebra homorphism between an algebra of holomorphic functions and an algebra of bounded linear operators. This boundedness property depends on the operator and on the Banach space, and is highly unstable under perturbations. It does, however, have some interesting characterisations in terms of square function estimates (operator theoretic generalisations of certain unconditionality properties of Fourier series). In concrete cases of differential operators, these square function estimates can often be established by harmonic analytic methods.

In this talk, after presenting the general theory, we will focus on differential operators, showing that there is much to gain from using generalisations of the conical square functions of Hardy space theory instead of the standard square functions of Littlewood-Paley theory. We will consider the connection with the current effort in harmonic analysis to handle singular integral operators with rough kernels, and explore, very briefly, some applications to stochastic and deterministic PDE. The emphasis, however, will be on the operator theory, and on how an abstract version of heat kernel bounds interacts with conical square functions in the same way that the notion of R-boundedness interacts with standard square functions. The talk is based on joint works with P. Auscher, D. Frey, C. Kriegler, A. McIntosh, S. Monniaux, and J. van Neerven.

Denis Potapov, Sydney, Australia.

Title: ***Recent successes in perturbation theory.***

Abstract: I will talk about a series of significant advanced in the perturbation theory thanks to discovery of new methods in noncommutative analysis and double operator theory. This includes, the resolution of M.G Krein conjecture on operator Lipschitz functions; discovery of the higher order spectral shift function and the trace formula.

Dan Timotin, Bucharest, Romania.

Title: ***An extremal problem for characteristic functions.***

Abstract: We discuss a dual H^1/H^∞ extremal problem for characteristic functions on the unit circle. We give general estimates and provide explicit solutions in certain particular cases. This is a joint work with I. Chalendar, S.R. Garcia, and W.T. Ross.

Victor Vinnikov, Be'er Sheva, Israrel

Title: ***Transfer-function realization and zero/pole structure for multivariable rational matrix functions: the converse analysis.***

Abstract: Reconstruction of a nondegenerate matrix-valued rational function of a single complex variable from its zero and pole data, including the directional information, has been studied at length by Israel Gohberg and collaborators in the 1980s. The solution appears as the transfer function of an input/state/output linear system. This talk will present an ongoing journey, joint with Quanlei Fang and Joe Ball, towards extending these ideas to the multivariable case, namely, a nondegenerate matrix-valued rational function of d complex variables. Zeroes and poles are now algebraic hypersurfaces in the d-dimensional affine space $\mathbb{C}^d$ (or in its projective compactification $\mathbb{P}^d$), their directions are vector bundles, or more general torsion free sheaves, on these hypersurfaces, and the solution appears as the transfer function of a Fornasini–Marchesini multidimensional linear systems. The companion lecture of Joe Ball at this IWOTA focused on the direct analysis of zero and pole varieties and sheaves of a given nondegenerate multivariable rational matrix-valued function. This lecture will pursue the converse analysis of constructing a Fornasini–Marchesini realization of a function with a given zero and pole data. We will pay a special attention to the case $d = 2$ where one can profitably use a well developed theory of determinantal representations of plane algebraic curves. Even

the case of scalar functions is interesting, giving new insights into minimality properties of multivariable realizations.

Nicholas Young, Leeds and Newcastle, UK.

Title: ***Operator monotone functions and Löwner functions of several variables.***

Abstract: This is joint work with Jim Agler (UCSD) and John E. McCarthy (Washington University).

A famous theorem of Karl Löwner in the 1930s asserts that a real-valued function f on a real interval (a, b) acts monotonically on self-adjoint matrices if and only if f extends to an analytic function on the upper halfplane Π that maps Π to itself. We prove generalizations of Löwner's result to several variables. We give a characterization of functions of d variables that are locally monotone on d-tuples of commuting self-adjoint n-by-n matrices. We also characterize all rational functions of two variables that are operator monotone in a rectangle.

Our paper with the above title appeared in *Annals of Mathematics* **176** (2012) 1783–1826.

Ajit Iqbal Singh, Indian Statistical Institute, Delhi.

Abstract: It all started with Einstein, Podolsky and Rosen (EPR) in 1930s. Holevo explained and developed well the mathematical theory of quantum communication channels in early 1970s. A magical physical rebirth of EPR (maximally entangled) states and channels was well-managed by Bennett, Brassard, Crepeau, Josza, Peres, Wiesner and Wootters in early 1990s. The tree has grown ever since, thanks to all of them and likes of Knill, Shor, Werner, Winter et al. A quantum channel is a completely positive trace preserving map (Schroedinger picture) and its dual is a completely positive unital map (Heisenberg picture). This talk aims at giving basics through examples and displaying the recent use of underlying tools such as Dvoretzkys Theorem, particularly Milmans proof by Winter and Hayden in constructing counterexamples to Maximal p-norm multilplicativity conjecture. It draws upon Parthasarathys lucid books, Winters excellent talks at the AQIS-2013 and discussion with them.