Operators, Functions and Linear Systems

Organizers: **S. ter Horst and M.A. Kaashoek**

Daniel Alpay, Ben-Gurion University of the Negev

Title: Schur analysis in the setting of slice hyper-holomorphic functions

Abstract: In the setting of Clifford algebras and quaternions, there are at least two extensions of the notion of analyticity, Fueter series and slice hyper-holomorphic functions. This last notion is of particular importance since it relates to the functional calculus of N possibly non-commuting operators on a real Banach space. In the talk we report on recent joint work (see [1]–[4]) with Fabrizio Colombo and Irene Sabadini (Politecnico Milano), where the notion of functions analytic and contractive in the open unit disk are replaced by functions slice hyper-holomorphic in the open unit ball of the quaternions and bounded by one in modulus there. We outline the beginning of a Schur analysis in this setting.

[1] D. Alpay, F. Colombo and I. Sabadini. Schur functions and their realizations in the slice hyperholomorphic setting. Integral Equations and Operator Theory, vol. 72 (2012), pp. 253-289.

[2] D. Alpay, F. Colombo and I. Sabadini. Krein-Langer factorization and related topics in the slice hyperholomorphic setting. Journal of Geometric Analysis. To appear.

[3] D. Alpay, F. Colombo and I. Sabadini. Pontryagin de Branges-Rovnyak spaces of slice hyperholomorphic functions. Journal d'Analyse Mathématique. To appear.

[4] D. Alpay, F. Colombo and I. Sabadini. On some notions of convergence for n-tuples of operators. Mathematical Methods in the Applied Sciences. To appear.

Joseph A. Ball, Virginia Tech, Blacksburg

Title: Convexity analysis and integral representations for generalized Schur/Herglotz function classes

Abstract: We report on recent work with M.D. Guerra Huamán characterizing the extreme points of the compact convex set of normalized matrix-valued Borel measures on a compact Hausdorff space subject to finitely many linear constraints. We indicate how this result leads to integral-representation formulas for functions in the matrix-valued Schur-Agler class in the general test-function setting of Dritschel-Marcantognini-McCullough. Particular special cases of interest include the Schur class over a finitely connected planar domain, as well as the Neil algebra (equivalently, the constrained H^∞ algebra over the unit disk), where we recover the explicit results of Dritschel-Pickering. The Schur-Agler class over the polydisk in principle fits the same framework, but unfortunately the associated normalized class of measures involves an infinite collection of side linear constraints.

Snehalatha Ballamoole, Mississippi State University, USA

Title: A class of integral operators on spaces of analytic functions

Abstract: We determine the spectral properties as well as resolvent estimates for a class of integral operators $T_{\mu,\nu} f(z) = z^{\mu-1}(1-z)^{-\nu}\int_0^z f(\xi)\xi^{-\mu}(1-\xi)^{\nu-1}\,d\xi$ acting on either the analytic Besov spaces or other Banach spaces of analytic functions on the unit disc, including the classical Hardy and weighted Bergman spaces as well as certain Dirichlet spaces and generalized Bloch spaces. Our

results unify and extend recent work by Aleman and Persson [2], the current authors, and Albrecht and Miller [1].

This is joint work with Len and Vivien Miller.

[1] E. Albrecht and T. L. Miller. Spectral properties of two classes of averaging operators on the little Bloch space and the analytic Besov spaces. *Complex Analysis and Operator theory, DOI 10.1007/s11785-012-0279-x.*

[2] A. Aleman and A.-M. Persson, Resolvent estimates and decomposable extensions of generalized Cesàro operators, *J. Funct. Anal. 258* (2010), 67-98.

[3] S. Ballamoole, T. L. Miller and V. G. Miller, Spectral properties of Cesàro-like operators on weighted Bergman spaces, *J. Math. Anal. Appl. 394* (2012,) 656-669.

Alon Bulbil, Ben-Gurion University of the Negev

Title: Continuous Stochastic Linear Systems

Abstract: In the work of Ariel Pinhas and Prof. Daniel Alpay, a new approach to discrete linear stochastic systems, using the white noise space and the theory of stochastic distributions was developed. In this talk we study continuous linear systems using the above approach, in particular we focus on the following questions:

1)The state space equations of continuous stochastic systems,

$$x^{'} = Ax + Bu$$
$$y = Cx + Du$$

where A, B, C, D are matrices with entries in K.
Here K denotes the $Kondratiev$ space of stochastic distributions.

2)Observability.

3)Systems given in descriptor form, the case where the state operator is not bounded.

$$Ex^{'} = Ax + Bu$$
$$y = Cx + Du$$

E, A, B, C, D are matrices with entries in K.

4)The infinite case $l_2 \bigotimes K$.

We also study the time varying case where the operators depend on t.

Santanu Dey, Indian Institute of Technology Bombay, Powai -400076

Title: Functional Model for multi-analytic operators

Abstract: We study a functional model for any multi-analytic operator Θ. We establish a condition in terms of certain spaces in this model which is equivalent to injectivity of the Θ. We show that an application of our functional model yields that the characteristic function of a reduced lifting is injective. We also prove the converse, i.e., if a given contractive multi-analytic function is injective then it can be realized as the characteristic function of a reduced lifting. We study our model for the special case of Schur functions.

Roland Duduchava, A. Razmadze Mathematical Institute, I. Javakhishvili Tbilisi State University, Tbilisi, Georgia

Title: Calculus of Gunter's derivatives and a shell theory

Abstract: There exist number of approaches proposed for modeling linearly elastic flexural shells (see papers by Cosserats (1909), Goldenveiser (1961), Naghdi (1963), Vekua (1965), Novozhilov (1970), Koiter (1970)). We suggest a different approach based on a calculus of Gunter's and Stock's derivatives on a hypersurface $\mathcal{S}$ be given by a local immersion $\Theta : \Omega \to \mathcal{S} \subset \mathbb{R}^n$, $\Omega \subset \mathbb{R}^{n-1}$. Differential operators on the surface are represented in terms of Günter's derivatives $\mathcal{D}_j := \partial_j - \nu_j(x)\partial_\nu$, $j = 1, \dots, n$, where $\nu = (\nu_1, \dots, \nu_j)^\top$ is the outer unit normal vector to $\mathcal{S}$ and $\partial_\nu := \sum_{j=1}^n \nu_j \partial_n$ is the normal derivative. We write the surface gradient $\nabla_\mathcal{S}$, the surface divergence $\mathrm{Div}_\mathcal{S}$, the Laplace-Beltrami operator $\mathbf{\Delta}_\mathcal{S}$, the Lamé operator $\mathcal{L}_\mathcal{S}$, the deformation tensor $(\mathrm{Def}_\mathcal{S}(U)$, the covariant derivative $\nabla^\mathcal{S}_W U$ (the Levi-Civita connection) in terms of Günter's derivatives in rather compact forms, very similar to their represntation in cartesian coordinates of the ambient Euclidean space $\mathbb{R}^n$.

Concerning an application to shell theory: shell is minded as a "tubular" neighborhood $\Omega^\varepsilon := \Big\{x \in \mathbb{R}^n \,:\, x = x + t\boldsymbol{\nu}(x) = \Theta(y) + t\boldsymbol{\nu}\big(\Theta(y)\big)\,, \quad y \in \omega\,,\ -\varepsilon \leqslant t \leqslant \varepsilon\Big\}$ of the middle surface $\mathcal{S}$. Shell equation is derived from the 3D Lamé equation by a formal asymptotic analysis. The derived equation is written in Gunter's derivatives and has rather compact form (see [1]). The final goal of the investigation is to derive 2D shell equations, obtained by formal asymptotic analysis, by Γ-Convergence as in [2].

1. R. Duduchava, A revised asymptotic model of a shell. *Memoirs on Differential Equations and Mathematical Physics* **52**, 2011, 65-108.

2. G. Friesecke, R.D. James, S. Müller, A theorem on geometric rigidity and the derivation of nonlinear plate theory from three dimensional elasticity, *Communications on Pure and Applied Mathematics* bf 55, 11, 2002, 1461–1506.

Mehdi Ghasemi, Nanyang Technological University

Title: Moment problem for continuous linear functionals

Abstract: Let A be a commutative unital algebra over real numbers and let τ be a locally multiplicatively convex topology on A. We apply T. Jacobi's representation theorem to determine the closure of a $\sum A^{2d}$-module S of A in the τ-topology, for any integer $d \geq 1$. We show that this closure is exactly the set of all elements $a \in A$ such that $\alpha(a) \geq 0$ for every τ-continuous $\mathbb{R}$-algebra homomorphism $\alpha : A \to \mathbb{R}$ with $\alpha(S) \subseteq [0, \infty)$. We obtain a representation of any linear functional $L : A \to \mathbb{R}$ which is continuous with respect to any such τ and nonnegative on S as integration with respect to a unique Radon measure on the space of all real valued $\mathbb{R}$-algebra homomorphisms on A.

Sanne ter Horst, North-West University, South Africa

Title: Equivalence after extension and Schur coupling coincide, on separable Hilbert spaces

Abstract: Two Banach space operators $U : \mathcal{X}_1 \to \mathcal{X}_2$ and $V : \mathcal{Y}_1 \to \mathcal{Y}_2$ are said to be (a) *equivalent after extension* if there exist Banach spaces $\mathcal{X}_0$ and $\mathcal{Y}_0$ such that $U \dotplus I_{\mathcal{X}_0}$ and $V \dotplus \mathcal{Y}_0$ are equivalent and (b) *Schur coupled* in case there exists an operator matrix $\left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right] : \left[\begin{smallmatrix} \mathcal{X}_1 \\ \mathcal{Y}_1 \end{smallmatrix}\right] \to \left[\begin{smallmatrix} \mathcal{X}_2 \\ \mathcal{Y}_2 \end{smallmatrix}\right]$ with A and D invertible and

$$U = A - BD^{-1}C, \quad V = D - CA^{-1}B.$$

Bart and Tsekanovskii [2,3] studied the relation between these two notions, and the notion of *matricial coupling* which coincides with equivalence after extension, and proved that Schur coupling implies equivalence after extension. The converse question, whether matricial coupling implies Schur coupling, was answered affirmatively for matrices in (3) and later for Fredholm operators on Hilbert spaces [1], but the general case remained open. We prove that equivalence after extension implies Schur coupling in case (i) U or V can be approximated in norm by invertible operators or (ii) U or V is Moore-Penrose invertible. These two results together provide an affirmative answer to the question in case U and V are Hilbert space operator acting between separable Hilbert spaces. The talk is based on [4].

(1) H. Bart, I. Gohberg, M.A. Kaashoek, and A.C.M. Ran, Schur complements and state space realizations, *Linear Algebra Appl.* **399** (2005), 203–224.

(2) H. Bart and V.E. Tsekanovskii, Matricial coupling and equivalence after extension, in: *Operator Theory and Complex Analysis*, OT **59**, 1992, pp. 143-160.

(3) H. Bart and V.E. Tsekanovskii, Complementary Schur complements, *Linear Algebra Appl.* **197** (1994) 651-658.

(4) S. ter Horst and A.C.M. Ran, Equivalence after extension and matricial coupling coincide with Schur coupling, on separable Hilbert spaces, *Linear Algebra Appl.* **439** (2013), 793–805.

Jacob Jaftha, University of Cape Town

Title: Dissipative linear relations in Banach spaces and a multivalued version of the Lumer-Phillips Theorem

Abstract: The aim is to establish a Lumer-Phillips like characterisation of the generator of degenerate C-semigroups of contractions in Banach spaces. In this case the generator is a linear relation or multivalued map.

Salma Kuhlmann, University of Konstanz

Title: Application of Jacobi's Representation Theorem to locally multiplicatively convex topological real algebras

Abstract: Let A be a commutative unital real algebra and let ρ be a seminorm on A which satisfies $\rho(ab) \leq \rho(a)\rho(b)$. We apply T. Jacobi's representation theorem to determine the closure of a module S of A in the topology induced by ρ. We show that this closure is exactly the set of all elements $a \in A$ such that $\alpha(a) \geq 0$ for every ρ-continuous real algebra homomorphism $\alpha : A \to \mathbb{R}$ with $\alpha(S) \subseteq [0, \infty)$, and that this result continues to hold when ρ is replaced by any locally multiplicatively convex topology τ on A. We obtain a representation of any linear functional $L : A \to \mathbb{R}$ which is continuous with respect to any such ρ or τ and non-negative on S as integration with respect to a unique measure on the space of all real valued real-algebra homomorphisms on A, and we characterize the support of the measure obtained in this way.

This is a joint work with M. Ghasemi and M. Marshall.

Zinaida A. Lykova, Newcastle University

Title: 3-extremal holomorphic maps and the symmetrised bidisc

Abstract: We analyse the 3-extremal holomorphic maps from the unit disc $\mathbb{D}$ to the symmetrised bidisc $\mathcal{G} \stackrel{\text{def}}{=} \{(z+w, zw) : z, w \in \mathbb{D}\}$ with a view to the complex geometry and function theory of $\mathcal{G}$. These are the maps whose restriction to any triple of distinct points in $\mathbb{D}$ yields interpolation data that are only just solvable. We find a large class of such maps; they are rational of degree at most 4. It is shown that there are two qualitatively different classes of rational $\mathcal{G}$-inner functions of degree at most 4, to be called *aligned* and *caddywhompus* functions; the distinction relates to the cyclic ordering of certain associated points on the unit circle. The aligned ones are 3-extremal. We describe a method for the construction of aligned rational $\mathcal{G}$-inner functions; with the aid of this method we reduce the solution of a 3-point interpolation problem for aligned holomorphic maps from $\mathbb{D}$ to $\mathcal{G}$ to a collection of classical Nevanlinna-Pick problems with mixed interior and boundary interpolation nodes. Proofs depend on a form of duality for $\mathcal{G}$. The talk is based on a joint work with Jim Agler and N. J. Young.

Sourav Pal, Department of Mathematics, Ben-Gurion University of the Negev, Israel.

Title: Spectral sets and distinguished varieties in the symmetrized bidisc

Abstract: We show that for every pair of matrices (S, P), having the closed symmetrized bidisc Γ as a spectral set, there is a one dimensional complex algebraic variety Λ in Γ such that for every matrix valued polynomial $f(z_1, z_2)$,

$$\|f(S,P)\| \le \max_{(z_1,z_2)\in\Lambda} \|f(z_1,z_2)\|.$$

The variety Λ is shown to have the determinantal representation

$$\Lambda = \{(s,p) \in \Gamma : \det(F + pF^* - sI) = 0\},$$

where F is the unique matrix of numerical radius not greater than 1 that satisfies

$$S - S^*P = (I - P^*P)^{\frac{1}{2}} F (I - P^*P)^{\frac{1}{2}}.$$

When (S, P) is a strict Γ-contraction, then Λ is a *distinguished variety* in the symmetrized bidisc, i.e. a one dimensional algebraic variety that exits the symmetrized bidisc through its distinguished boundary. We characterize all distinguished varieties of the symmetrized bidisc by a determinantal representation as above.

Denis Potapov, UNSW Australia

Title: Frechet differentiability of Schatten-von Neumann p-norms

Abstract: One of the open questions in the theory of Schatten ideals S^p of compact operators is whether their norms have the same differentiability properties as the norms of their commutative counterparts. The talk will answer the question positively. The solution is based on earlier resolution of L.S. Koplienkos conjecture concerning existence of higher order spectral shift functions.

Alexander Sakhnovich, University of Vienna

Title: Explicit solutions of linear and nonlinear evolution equations depending on several variables

Abstract: We develop the generalized Bäcklund-Darboux transformation (GBDT) approach (see review [1] and references therein) for the case of linear systems depending on k ($k > 1$) variables and nonlinear integrable equations depending on $2+1$ variables. The cases of linear non-stationary Schrödinger and Dirac equations and nonlinear Kadomtsev-Petviashvili and Davey-Stewartson equations are considered in greater detail.

[1] A.L. Sakhnovich. On the GBDT version of the Bäcklund–Darboux transformation and its applications to the linear and nonlinear equations and spectral theory. *Mathematical Modelling of Natural Phenomena* **5**: 340–389 (2010).

Eli Shamovich, Ben Gurion University of the Negev

Title: Determinantal Representations and Hyperbolicity on the Grassmannian

Abstract: In this talk I will present joint work with Victor Vinnikov. A Livsic-type determinantal representation is a tensor $\gamma \in \wedge^{k+1}\mathbb{C}^{d+1} \otimes M_n(\mathbb{C})$. We define a subvariety of $\mathbb{P}^d$ associated to γ by:

$$D(\gamma) = \left\{ \mu \in \mathbb{P}^2 \mid \exists\, v \in \mathbb{C}^n \setminus 0\,, (\gamma \wedge \mu)v = 0. \right\}.$$

Here we consider $\gamma \wedge \mu$ as a mapping from $\mathbb{C}^n$ to $\wedge^{k+2}\mathbb{C}^{d+1} \otimes C^n$. In other words v is in the intersection of kernels of the coefficient of the basis vectors of $\wedge^{k+2}\mathbb{C}^{d+1}$ in $\gamma \wedge \mu$. We will say that a variety X of dimension k admits a determinantal representation, if there exists a tensor γ as above, such that $X = D(\gamma)$ as sets.

The term Livsic-type determinantal representation originates in the works of Moshe Livsic and his collaborators. They've studied joint spectra of pairs of non-selfadjoint operators with finite dimensional imaginary parts and associated overdetermined systems. Their study naturally led to determinantal representations of plane projective curves. Later Victor Vinnikov showed that every projective plane curve admits such a representation. The study of general Livsic-type determinantal representations arose in our work on joint spectra of d-tuples of commuting non-selfadjoint operators and associated systems.

Joe Ball and Victor Vinnikov introduced an algorithm that gives a determinantal representation to plane curves with at most node-type singularities. We will generalize this algorithm to provide "good" determinantal representations to curves in $\mathbb{P}^d$.

Then we will introduce the notion of hyperbolicity for hypersurfaces on the Grassmaniann. We will show how this notion relates to the classical notion of hyperbolic hypersurface and how it is different. We will show that the aforementioned algorithm allows to connect the two notions in the case of projective curves.

Victor Vinnikov, Ben Gurion University of the Negev

Title: Determinantal representations of stable and hyperbolic polynomials

Abstract: A representation of a polynomial in several variables as the determinant of a matrix of linear forms with the coefficient matrices having certain metric properties (such as being selfadjoint, unitary, or contractive) can witness appropriate stability properties of the polynomial. One example is provided by positive selfadjoint determinantal representations of homogeneous hyperbolic polynomials, that play a key role in semidefinite programming. Another example is that of contractive determinantal representations of polynomials that are stable (with respect to the unit polydisc), that are intimately related to multidimensional conservative realizations and to the multivariable von Neumann inequality. In both cases little is known for more than two variables. I will discuss new proofs

of the results for polynomials in two variables (or homogeneous polynomials in three variables) that are based on standard matrix-valued one variable factorization results on the real line or on the unit disc. This approach is particularly interesting in the case of homogeneous hyperbolic polynomials in three variables, yielding (a somewhat weakened form of) a 1958 conjecture of Lax that was originally established by algebro-geometric techniques.

Nicholas Young, Leeds and Newcastle Universities
Title: Realization of symmetric analytic functions of noncommuting variables
This is joint work with Jim Agler.
Abstract: We study symmetric analytic nc-functions on the *biball*

$$B^2 \stackrel{\text{def}}{=} \bigcup_{n=1}^{\infty} \mathbb{B}_n \times \mathbb{B}_n,$$

where $\mathbb{B}_n$ denotes the open unit ball of the space $\mathcal{M}_n$ of $n \times n$ complex matrices. B^2 is the non-commutative analogue of the bidisc. We show that every such function that is bounded by 1 in norm factors through a certain nc-domain Ω in the space

$$\mathcal{M}^{\infty} \stackrel{\text{def}}{=} \bigcup_{n=1}^{\infty} \mathcal{M}_n^{\infty}.$$

Here an *nc-function* is a function defined on an nc-domain Ω in $\bigcup_{n=1}^{\infty} \mathcal{M}_n^d$ (for some $d \leq \infty$) that respects direct sums and similarities and maps $\Omega \cap \mathcal{M}_n^d$ to $\mathcal{M}_n$. An *nc-domain* is a domain in $\bigcup_{n=1}^{\infty} \mathcal{M}_n^d$ that is closed under direct sums and unitary similarity.

More precisely, there exists an nc-domain Ω in $\mathcal{M}^{\infty}$ and an analytic nc-function $S : B^2 \to \Omega$ (given by a simple rational expression) with the following property. For every symmetric nc-function φ on B^2 that is bounded by 1 in norm there exists an analytic nc-function Φ on Ω such that $\varphi = \Phi \circ S$; moreover Φ can be expressed by means of a non-commutative version of the familiar linear fractional realization formula for functions in the Schur class.

Geometry of Banach spaces in Operator Theory

Organizer: **T.S.S.R.K. Rao**

S. Dutta (Indian Institute of Technology, Kanpur)

Title: Predual of completely bounded multipliers.

Abstract: The space of $L_p(G)$-multipliers $1 < p < 2$, on locally compact group G is usually studied through it's pre-dual $A_p(G)$. Interesting conclusions on the group G itself can be derived from the properties of $A_p(G)$. However, if we change the category of Banach spaces to *Operator spaces*, story becomes quite different.

We consider canonical operator space structure on $L^p(G)$ through operator space complex interpolation *a la* G. Pisier.

In this talk we describe pre-dual of completely bounded $L_p(G)$-multipliers.

V. Indumathi (Pondicherry University, Pondicherry)

Title: Polyhedral conditions and Best Approximation Problems.

Abstract: Study of proximinality and semi-continuity of metric projections onto subspaces of finite codimension in the last two decades have established close links between these problems and the geometric properties of Banach spaces related to polyhedral conditions. Best approximation results, related to subspaces of finite codimension, derived using geometric properties of the space are discussed.Open problems related to possible bearing of the polyhedral conditions on the relatively new notion of "Ball proximinality" are presented.

Lajos Molnar (University of Debrecen, Hungary)

Title: Isometries of certain nonlinear spaces of matrices and operators.

Abstract: We describe the structure of all surjective isometries of the unitary group and that of the cone of positive definite matrices with respect to various metrics that are related to unitarily invariant norms and geodesic distances. Next we determine the surjective isometries of the set of all rank-n projections on a Hilbert space equipped with the gap metric. In the particular case where $n = 1$, our result reduces to Wigner's famous theorem on the structure of quantum mechanical symmetry transformations. The latter result is joint with J. Jamison and F. Botelho.

Ashoke. K. Roy (Ramakrishna Mission Vivekananda University, Howrah)

Title: On Silov boundary for function spaces.

Abstract: In this paper we develop the notion of Silov boundary for complex normed linear spaces, for which 0 is not a weak*- accumulation point of the extreme points of the dual unit ball. We give an example to show that the set of peak points need not be a boundary. We show that the closure of the Choquet boundary is the Silov boundary and also give a proof of the Bishop's theorem that describes the Choquet boundary, in the case of subalgebras of continuous functions that do not contain the

constant function.

Jiří Spurný (Charles University, Czech Republic)

Title: Baire classes of Banach Spaces and C^* Algebras.

Abstract: Let X be a real or complex Banach space and let B_{X^*} stand for its dual unit ball endowed with the weak* topology. Then X is isometrically embedded into the space of continuous functions on B_{X^*} via the canonical embedding. We recall the definitions of Baire classes of the second dual X^{**} of X from [1]. Let $X_0^{**} = \{x|_{B_{X^*}} : x \in X\}$,

$$X_1^{**} = \{x^{**}|_{B_{X^*}} : x^{**} \in X^{**}, x^{**} \text{ is a weak}^* \text{ limit of a sequence } (x_n) \text{ in } X\},$$

and for $\alpha \in (1, \omega_1)$, let X_α^{**} consist of weak* limits of sequences contained in $\bigcup_{\beta<\alpha} X_\beta^{**}$. The spaces X_α^{**} are called the *intrinsic α-Baire classes* of X^{**}.

Further, the α-th *Baire class* of X^{**} is defined as

$$X_{\beta_\alpha}^{**} = \{x^{**}|_{B_{X^*}} : x^{**} \in X^{\perp\perp}, x^{**}|_{B_{X^*}} \text{ is of Baire class } \alpha\}.$$

Given an element $x^{**} \in X^*$, it can be verified that $x^{**}|_{B_{X^*}} \in X_{\beta_\alpha}^{**}$ if and only if $x^{**}|_{B_{X^*}}$ is of Baire class α and $x^{**}|_{B_{X^*}}$ satisfies the barycentric calculus, i.e.,

$$x^{**}\left(\int_{B_{X^*}} \text{id } d\mu\right) = \int_{B_X^*} x^{**} d\mu$$

for every probability measure $\mu \in \mathcal{M}^1(B_{X^*})$.

It is easy to see that $X_\alpha^{**} \subset X_{\beta_\alpha}^{**}$. By the Mokobodzki theorem, $X_1^{**} = X_{\beta_1}^{**}$ (see [1, Theorem II.1.2(a)]). For higher classes, there is a striking example of Talagrand (see [5, Theorem]) who constructed a separable Banach space X such that $\bigcup_{\alpha<\omega_1} X_\alpha^{**} = X_0^{**}$ and $X_{\beta_2}^{**}\, X_0^{**} \neq \emptyset$.

A Banach space X is called an L_1-predual if X^* is isometric to a space $L^1(X, \mathcal{S}, \mu)$ for some measure space $(X, \mathcal{S}, \mu)$. The aim of the talk will be a survey of results from [4], [2], [3] on relations between Baire classes and intrinsic Baire classes of L_1-preduals and C^*-algebras.

Vrej Zarikian (U.S. Naval Academy)

Title: Bimodules over Cartan Subalgebras, and Mercer's Extension Theorem. Abstract: In a 1991 paper, Mercer asserts the following extension theorem:

> *For $i = 1, 2$, let $\mathcal{M}_i$ be a von Neumann algebra with separable predual, let $\mathcal{D}_i \subseteq \mathcal{M}_i$ be a Cartan subalgebra, and let $\mathcal{D}_i \subseteq \mathcal{A}_i \subseteq \mathcal{M}_i$ be a σ-weakly-closed non-self-adjoint algebra which generates $\mathcal{M}_i$. If $\theta : \mathcal{A}_1 \to \mathcal{A}_2$ is an isometric algebra isomorphism such that $\theta(\mathcal{D}_1) = \mathcal{D}_2$, then there exists a unique $*$-isomorphism $\phi\theta : \mathcal{M}_1 \to \mathcal{M}_2$ such that $\phi\theta|_{\mathcal{A}_1} = \theta$.*

Mercer's proof relies on the *Spectral Theorem for Bimodules* of Muhly, Saito, and Solel (hereafter STB), which characterizes the σ-weakly-closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$ in terms of measure-theoretic data. Unfortunately, both proofs of STB in the literature contain gaps, and so the validity of STB, and therefore of Mercer's extension theorem, is unclear. In this talk, based on joint work with Jan Cameron (Vassar) and David Pitts (Nebraska), we prove Mercer's extension theorem under the additional hypothesis that θ is σ-weakly continuous. Our argument makes use of ideas from operator space theory, as well as a characterization of the Bures-closed $\mathcal{D}_i$-bimodules of $\mathcal{M}_i$, and does not require that $\mathcal{M}_i$ have separable predual.

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Concrete Operators

Organizers: **Isabelle Chalendar, Alfonso Montes Rodriguez & Ilya Spitkovsky**

Gerardo Chacon (Pontificia Universidad Javeriana, Bogata, Colombia)
Title: Composition Operators and derivative-free characterizations of Dirichlet-type Spaces
Abstract: Let μ be a positive finite Borel measure on the unit circle $\partial\mathbb{D}$, and let P_μ be its harmonic extension to the unit disc $\mathbb{D}$, i.e.,

$$P_\mu(z) = \int_{\partial\mathbb{D}} \frac{1-|z|^2}{|\zeta - z|^2}\, d\mu(\zeta), \qquad (z \in \mathbb{D}).$$

The Dirichlet-type space $D(\mu)$ is defined as the set of holomorphic functions f for which

$$\int_{\mathbb{D}} |f'(z)|^2\, P_\mu(z)\, dA(z) < \infty.$$

We will review some properties of composition operators acting on this spaces as well as some characterizations and an atomic decomposition of the space.

The talk is partially based on a joint work with Xiaosong Liu and Zengjian Lou.

Gopal Datt (University of Delhi, India)
Title: Hankel to weighted Hankel operators
Abstract: TBA

George Exner (Bucknell University, USA)
Title: TBA
Abstract: TBA

Mubariz T. Garayev (King Saud University, Riyadh, Saudi Arabia)
Title: Some Concrete Operators and their Properties
Abstract: We consider integration and double integration operators, Hardy operator, multiplication and composition operators on the Lebesgue space $L_p[0,1]$ and Sobolev spaces $W_p^{(n)}[0,1]$ and $W_p^{(n)}([0,1]\times[0,1])$, and study their spectral properties. In particular, we calculate norm and spectral multiplicity of the Hardy operator and some multiplication operator, investigate its extended eigenvectors, characterize some composition operators in terms of the extended eigenvectors of Hardy operator, calculate the numerical radius of the integration operator on the real $L_2[0,1]$ space. The main method for our investigation is the so-called Duhamel products method introduced and used firstly by Wigley [1,2]. Some other questions are also discussed and posed.

Gyorgy Pal Geher (Bolyai Institute of Mathematics, University of Szeged, Hungary)
Title: Tree-shift Operators and their Cyclic Properties

Abstract: In this talk we consider a natural generalization of the so called (bounded) weighted bilateral, unilateral and backward shift operators with respect to an orthonormal basis in a complex Hilbert space $\mathcal{H}$. A bounded shift operator can be represented by a graph with weights on its vertices. For instance, let us consider a bounded weighted bilateral shift W defined by the equation $We_j = w_{j+1}e_{j+1}$, where $\{e_j\}_{j=-\infty}^{\infty}$ is an orthonormal basis. Then W can be represented by the graph for which the set of vertices is $\mathbb{Z}$, the set of directed edges is $\{(j, j+1) \in \mathbb{Z}^2 \colon j \in \mathbb{Z}\}$ and w_j is the weight with respect to the vertex j. In this context, the action of W is as follows: we map the coordinates of a vector to the vertices, shift them along the edges, and finally multiply the jth coordinate by w_j for each $j \in \mathbb{Z}$ to get the image. Note that the directed graph of W is a special (infinite) directed tree.

The idea is that, instead of the previous very special directed tree, we take an arbitrary directed tree, put some weights on the vertices and follow a very similar procedure to the one described above. Here the orthonormal basis is now indexed by the set of vertices of the directed tree (which was $\mathbb{Z}$ for the bilateral case). In this way we get a tree-shift operator.

The aim of this talk is to show how cyclicity is connected to asymptotic behaviour of Hilbert space contractions (Foias–Sz.-Nagy theory) and illustrate it on contractive tree-shift operators. In the first part of the talk we calculate the so called asymptotic limit and isometric asymptote of a tree-shift contraction and that of the adjoint. Then, in the second part, we use the results of the first part and similarity properties to infer some cyclicity and non-cyclicity theorems.

We will also consider a characterization of cyclicity on the class of weighted backward shift operators with countable multiplicity.

Hocine Guediri (King Saud University, Riyadh, Saudi Arabia)

Title: The Bergman Space as a Banach Algebra

Abstract: Let $\mathbb{D}$ be the unit disk in the complex plane $\mathbb{C}$, and let $dA(z)$ be the Lebesgue measure on $\mathbb{D}$. The Lebesgue space of p-summable complex-valued functions is denoted by $L^p(\mathbb{D}, dA)$. The Bergman space $L^p_a(\mathbb{D})$ is the Banach subspace of $L^p(\mathbb{D}, dA)$ consisting of analytic functions, with norm: $||f||_p = \left(\int_{\mathbb{D}} |f(z)|^p dA(z)\right)^{\frac{1}{p}}$. The Duhamel product of two analytic functions in $\mathbb{D}$ is given by

$$(f \circledast g)(z) := \frac{d}{dz}\int_0^z f(z-t)g(t)dt = \int_0^z f'(z-t)g(t)dt + f(0)g(z).$$

Wigley elaborated at length this product and used it to provide a Banach algebra structure to the Hardy space $\mathcal{H}^p(\mathbb{D}), p \geq 1$. Moreover, he described its Maximal ideal space. Merryfield and Watson extended the matter to the context of vector-valued Hardy spaces of the polydisk. In the last decade the Duhamel product has been extensively explored by M.T. Karaev and his collaborators; and many applications of it have been well investigated.

In this work, using the Duhamel product, we provide a Banach algebra structure to the Bergman space $L^p_a(\mathbb{D})$, for $p > 1$, and carry out many interesting consequences. Moreover, we examine also the case of the Dirichlet space $\mathcal{D}$.

Romesh Kumar (University of Jammu, India)

Title: Composition Operators and Multiplication Operators on Banach Function Spaces

Abstract: Let X and Y be two non–empty sets and let $V(X, \mathbf{C})$ and $V(Y, \mathbf{C})$ be two topological vector spaces of complex valued functions on X and Y, respectively, under pointwise vector space

operations, where $\mathbf{C}$ denotes the field of all complex numbers. Suppose $T : Y \mapsto X$ is a mapping such that $f \circ T \in V(Y, \mathbf{C})$, whenever $f \in V(X, \mathbf{C})$. Define a composition transformation $C_T : V(X, \mathbf{C}) \mapsto V(Y, \mathbf{C})$ as

$$C_T f = f \circ T, \ f \in V(X, \mathbf{C}).$$

If C_T is continuous, then C_T is called a composition operator induced by T.

Further, let $\pi : X \mapsto \mathbf{C}$ be a function. Then the multiplication transformation $M_\pi : V(X, \mathbf{C}) \mapsto V(X, \mathbf{C})$ is defined as

$$M_\pi f = \pi.f, \ f \in V(X, \mathbf{C}).$$

A continuous multiplication linear transformation is called a multiplication operator induced by π.

There is a vast literature on composition operators, multiplication operators and their applications. The composition appear naturally in the study of isometries of most of the function spaces. They are widely used in the Ergodic theory and in determing the asymptotic properities of dynamical systems and differentiable equations. In this talk I will dicuss composition operators and multiplication operators on rearrangement invariant spaces and Banach Function spaces. This talk is based on my joint work with Yunan Cui, Henryk Hudzik, Rajeev Kumar and Herra Saini.

Aneesh M (Indian Institute of Technology, Kanpur, India)

Title: Supercyclicity and frequent hypercyclicity in the space of self-adjoint operators

Abstract. TBA

Patryk Pagacz (Jagiellonian University, Krakow, Poland)

Title: on wandering vectors for isometries and szego-measure properties

Abstract: The talk is based upon a joint work with Z. Burdak, M. Kosiek and M. Słociński. It concerns linear isometries V on a complex Hilbert space $\mathcal{H}$.

Let μ be a non-negative regular Borel measure on the unit circle Γ. We say that μ is a Szegö measure if for any Borel subset $\omega \subset \Gamma$ the inclusion $\chi_\omega L^2(\mu) \subset H^2(\mu)$ implies $\mu(\omega) = 0$. Here χ_ω denotes the characteristic function of the set ω and $H^2(\mu)$ denotes the closure in $L^2(\mu)$ of the algebra of all analytic polynomials.

We introduce the definition which connects an isometry with the Szegö measures.

Let $V \in \mathcal{B}(\mathcal{H})$ be an isometry and $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$ be a reducing decomposition for V. We say that $\mathcal{H}_1 \oplus \mathcal{H}_2$ is a *Szegö-type decomposition* if for each $x \in \mathcal{H}_1$ the elementary spectral measure μ_x is Szegö singular and each $x \in \mathcal{H}_2$ is a linear combination of vectors which elementary measures are Szegö.

For each isometry $V \in \mathcal{B}(\mathcal{H})$ we can consider three subspaces.

We define,

$$\mathcal{H}_p := \bigcap \{H_1 : \mathcal{H} \ominus H_1 \text{ reduce } V \text{ to a direct sum of unilateral and bilateral shifts}\}.$$

Secondary, for each isometry $V \in \mathcal{B}(\mathcal{H})$ there exists a minimal unitary extension $\widetilde{V} \in \mathcal{B}(\mathcal{K})$. Let us denote

$$\mathcal{M} = \{M \subset \mathcal{H} | V(M) \subset M, \quad \bigvee_{n \geq 0} \widetilde{V}^{*n}(M) = \mathcal{K}\}.$$

And, finally let $\mathcal{H}_0 := (\mathcal{H}_w)^\perp$, where $\mathcal{H}_w$ is the subspace spanned by wandering vectors for V.

For this subspaces we have:
Let $V \in \mathcal{B}(\mathcal{H})$ be an isometry. Then

- the subspaces $\mathcal{H}_p, \bigcap \mathcal{M}, \mathcal{H}_0$ are reducing,
- the decompositions $\mathcal{H} = \mathcal{H}_p \oplus \mathcal{H}_q = \bigcap \mathcal{M} \oplus (\bigcap \mathcal{M})^\perp = \mathcal{H}_0 \oplus \mathcal{H}_w$ are Szegö-type decompositions,
- there is $\mathcal{H}_0 \subset \bigcap \mathcal{M} \subset \mathcal{H}_p$.

In the case of a unitary operator, we have a well-known decomposition: Let $U \in \mathcal{B}(\mathcal{H})$ be a unitary operator and μ_x be an elementary spectral measure for $x \in \mathcal{H}$.

Let denote $\mathcal{H}_\sigma$ and $\mathcal{H}_a$ by the sets of all $x \in \mathcal{H}$ such that the measure μ_x is singular (absolutely continuous).

Thus $\mathcal{H}_a$ and $\mathcal{H}_\sigma$ are reducing subspaces for U. Moreover, $\mathcal{H} = \mathcal{H}_\sigma \oplus \mathcal{H}_a$.

We examine the relation between the above mentioned decompositions. First of all we show the equality in the case of unitary operator. Let $U \in \mathcal{B}(\mathcal{H})$ be a unitary operator. Then $\mathcal{H}_0 = \bigcap \mathcal{M} = \mathcal{H}_p$.

If additional, there are nonzero wandering vectors for U, then three considered decompositions are equal to the Lebesgue decomposition, i.e. $\mathcal{H}_0 = \bigcap \mathcal{M} = \mathcal{H}_p = \mathcal{H}_\sigma$.

Secondary, we give the counterexamples to the equalities $\mathcal{H}_0 = \bigcap \mathcal{M}$ and $\mathcal{H}_0 = \mathcal{H}_p$.

Herve Queffelec (Universite Lille-Nord-de-France, Lille, France)

Title: Approximation numbers of composition operators on the Dirichlet space

Abstract: Let $\varphi : \mathbb{D} \to \mathbb{D}$ be analytic (non-constant) and $C_\varphi(f) = f \circ \varphi$. It is known that the operator C_φ is always bounded on the Hardy space H^2, and NSC for its compactness or membership in Schatten classes were given in the nineties. More recently and more precisely (Li-Q-Rodriguez-Piazza, 2012), its approximation numbers $a_n(C_\varphi)$ were studied, and the following parameter emerged:

$$\beta(C_\varphi) = \liminf_{n\to\infty} [a_n(C_\varphi)]^{1/n}, \quad 0 \le \beta(C_\varphi) \le 1.$$

Note that

$$\beta(C_\varphi) > 0 \iff a_n(C_\varphi) \ge \; c\,r^n \text{ for some } r > 0.$$

$$\beta(C_\varphi) < 1 \iff a_n(C_\varphi) \le \; Cr^n \text{ for some } r < 1.$$

It was proved that $\beta(C_\varphi) > 0$ and that $\beta(C_\varphi) = 1 \iff \|\varphi\|_\infty = 1$.

As concerns the Dirichlet space $\mathcal{D}$, the situation is more intricate: first, C_φ is nots always bounded on $\mathcal{D}$. Second, it can be "very" compact on H^2 and bounded, but not compact, on $\mathcal{D}$. More specifically, the approximation numbers of C_φ on H^2 and $\mathcal{D}$ can behave very differently.

In that talk, we shall present some parallel results on those approximation numbers of C_φ on $\mathcal{D}$, like:

1. $\beta(C_\varphi) > 0$ and moreover $a_n(C_\varphi)$ can tend to 0 arbitrarily slowly.

2. $\beta(C_\varphi) = 1 \iff \|\varphi\|_\infty = 1$.

3. For somes cusp maps, we have $a_n(C_\varphi) \approx e^{-n/\log n}$ on H^2 while $a_n(C_\varphi) \approx e^{-\sqrt{n}}$ on $\mathcal{D}$.

If one compares with the H^2-case, the proofs often necessitate several additional ingredients.

This is joint work with P.Lefevre, D.Li, L.Rodriguez-Piazza.

Daniel Seco (Universitat autonoma the Barcelona, spain)

Title: Cyclicity in Dirichlet-type spaces and optimal polynomials

Abstract: For functions f in Dirichlet-type spaces over the disk we study how to determine constructively the optimal polynomials p_n of degree n, in terms of the norm of $p_n f - 1$, concentrating on the case when f is a cyclic function. We then give upper and lower bounds for the ratio of convergence of this norm to zero as n approaches ∞. Then we will introduce a few new results about similar spaces over the bidisk.

Ilya Spitkovsky (College ofWilliam and Mary, Virginia, USA)

Title: On the kernel and cokernel of some Toeplitz and Wiener-Hopf operators

Abstract: In this work, joint with T. Ehrhardt, we show that the kernel and/or cokernel of a block Toeplitz operator $T(G)$ are trivial if its matrix-valued symbol G satisfies the condition $G(t^{-1})G(t)^* = I_N$. As a consequence, the Wiener-Hopf factorization of G (provided it exists) must be canonical. Our setting is that of weighted Hardy spaces of the unit circle. We then extend this result to Wiener-Hopf operators on weighted Hardy spaces of the real line, and also Toeplitz operators on weighted sequence spaces.

František Štampach (Czech Technical University, Prague, Czech Republic)

Title: The characteristic function for infinite Jacobi matrices, the spectral zeta function, and solvable examples

Abstract: At the beginning a function called $\mathfrak{F}$ will be introduced with its basic algebraic properties. We use function $\mathfrak{F}$ to analyze spectral properties of operators given by a semi-infinite Jacobi matrix of certain type. First of all, we define a characteristic function of the Jacobi matrix in terms of $\mathfrak{F}$. One can show that the zero set of the characteristic function actually coincides with the point spectrum of the corresponding Jacobi operator. Moreover, many other important formulas can be written in terms of $\mathfrak{F}$, for example, eigenvectors, the Green function (especially the Weyl m-function).

As an illustration several solvable examples with concrete Jacobi operators are discussed such that the characteristic function is explicitly expressible in terms of special functions.

Further, we present a formula for the logarithm of $\mathfrak{F}$. Using this formula and considering Jacobi matrices with a finite Hilbert-Schmidt norm and vanishing diagonal we can express the spectral zeta function as a power series in the matrix entries. Again, the general formula will be applied in several concrete examples involving Bessel and q-Airy functions.

This work is a joint work with Pavel Šťovíček and it is partially based on the papers F. Štampach, P. Šťovíček: Linear Alg. Appl. 434 (2011) 1336-1353 and Linear Alg. Appl. 438 (2013) 4130-4155.

Functional and harmonic analytic aspects of wavelets and coherent states

Organizers: **S. Twareque Ali, J.-P. Antoine and J.-P. Gazeau**

S. Twareque Ali, Concordia University, Montreal, CANADA

Title: Quaternionic Wavelets and Coherent States

Abstract: We look at some properties of the quaternionic affine group, its representations on complex and quaternionic Hilbert spaces and the associated wavelet transforms.

J.-P. Antoine, Université Catholique de Louvain, Louvain-la-Neuve, BELGIUM

Title: Wavelets and multiresolution: from NMR spectroscopy to the analysis of video sequences

Abstract: We review the general properties of the wavelet transform, both in its continuous and its discrete versions, in one or two dimensions, and we describe some of its applications in signal and image processing. We also consider its extension to higher dimensions, to more general manifolds (sphere, hyperboloid,. . .) and to the space-time context, for the analysis of moving objects.

P.K. Das, I.S.I., Kolkata, India

Title: Generation of a superposition of coherent states in a resonant cavity and its nonclassicality and decoherence

Abstract: We discuss nonclassicality of superposition of coherent states in terms of sub-Poissonian photon statistics as well as the negativity of the Wigner function. We derive an analytic expression for the Wigner function from which we find that the function has some negative region in phase space. We obtain a compact form of the Wigner function when decoherence occurs and study the effect of decoherence on the state. We demonstrate the behaviour of the nonclassicality indicator.

A. Odzijewicz, University of Bialystok, Bialystok, POLAND

Title: Positive kernels and quantization

Abstract: TBA

Multivariable Operator Theory

Organizers: **Gadadhar Misra and Jaydeb Sarkar**

Sameer Chavan from IIT Kanpur, India

Title: Conditional completely hypercontractive tuples

Abstract: Motivated by some structural properties of the Drury-Arveson d-shift, we investigate a subclass of conditionally positive definite functions defined on the semi-group $\mathbb{N}$, and its operator theoretic counter-part which we refer to as the class of conditional completely hypercontractive tuples (for short, CCH tuples). We obtain a Lévy-Khinchin-type integral representation for the spherical generating tuples associated with CCH tuples and discuss its applications. For instance, under some mild integrability assumption on the associated Lévy measure, this integral representation can be used to locate the Taylor spectra of CCH tuples.

The talk is based on the joint work with V. M. Sholapurkar.

B. Krishna Das from ISI Bangalore, India

Title: Tensor product of quotient Hilbert modules

Abstract. In this talk, I will discuss a unified approach to problems of tensor product of quotient modules of Hilbert modules over $\mathbb{C}[z]$ and corresponding submodules of reproducing kernel Hilbert modules over $\mathbb{C}[z_1, \ldots, z_n]$ and the doubly commutativity property of module multiplication operators by the coordinate functions. More precisely, for a reproducing kernel Hilbert module $\mathcal{H}$ over $\mathbb{C}[z_1, \ldots, z_n]$ of analytic functions on the polydisc in $\mathbb{C}^n$ which satisfies certain conditions, we will see that any quotient module $\mathcal{Q}$ of $\mathcal{H}$ is doubly commuting if and only if $\mathcal{Q}$ is of the form $\mathcal{Q}_1 \otimes \cdots \otimes \mathcal{Q}_n$, for some one variable quotient modules $\{\mathcal{Q}_1, \ldots, \mathcal{Q}_n\}$. For $\mathcal{H}$ the Hardy module over polydisc $H^2(\mathbb{D}^n)$, this reduces to some recent results by Izuchi, Nakazi and Seto and J. Sarkar. This will also provide a classification of co-doubly commuting submodules for a class of reproducing kernel Hilbert modules over the unit polydisc and further insight into the wandering subspaces and ranks of co-doubly commuting submodules.

This is a joint work with A. Chattopadhyay and J. Sarkar.

Santanu Dey from IIT Mumbai, India

Title: Characteristic function of liftings

Abstract: Certain multi-analytic operators are shown to be complete unitary invariants for a large class liftings of row contractions called the reduced liftings. These are called characteristic functions of liftings. We also answer the converse of this. For this a functional model is developed. We also obtain a factorization result of the characteristic function and a transfer function realization for it.

Kalpesh Haria from IIT Mumbai, India

Title: Outgoing Cuntz scattering system for a coisometric lifting and transfer function

Abstract: We study a coisometry that intertwines Popescus presentations of minimal isometric dilations of a given operator tuple and of a coisometric lifting of the tuple. Using this we develop an outgoing Cuntz scattering system which gives rise to an input output formalism. A transfer function is introduced for the system. We also compare the transfer function and the characteristic function for the associated lifting.

Sanne ter Horst from North-West University, South Africa

Title: Stability of noncommutative multidimensional systems and structured Stein inequalities

Abstract: For an $n \times n$ matrix A, it is well known that stability of A, in the sense that $A^k x \to 0$ as $k \to \infty$ for any vector x, holds if and only if one of the following equivalent conditions is satisfied:

(i) $I - zA$ is invertible for all z in the closed unit disk $\overline{\mathbb{D}}$;

(ii) there exists an invertible $n \times n$ matrix S such that $\|S^{-1}AS\| < 1$;

(iii) there exists a positive definite solution X to the strict Stein equation $X - A^*XA > 0$.

In the context of certain nD-systems, the equivalence between appropriately modified (structured) versions of these three conditions fails, in particular the equivalence between the modifications of (i) and (ii), as was shown by Anderson, Agathoklis, Jury and Mansour in 1986. However, by an enhancement of the structure, one can arrive at a generalization of the above result with three statements that are equivalent to stability. We discuss this behavior in the context of the structured noncommutative multidimensional linear systems associated with the graph formalism setup studied by Ball-Groenewald-Malikorn.

The talk is based on joint work with Joe Ball and Gilbert Groenewald.

Il Bong Jung from Kyungpook National University, Korea

Title: On quadratically hyponormal weighted shifts

Abstract: Let $\mathcal{H}$ be a separable infinite dimensional complex Hilbert space and let $L(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. For $A, B \in L(\mathcal{H})$, we set $[A, B] := AB - BA$. A k-tuple $\mathbf{T} = (T_1, \cdots, T_k)$ of operators in $L(\mathcal{H})$ is called hyponormal if the operator matrix $([T_j^*, T_i])_{i,j=1}^k$ is positive on the direct sum of k copies of $\mathcal{H}$. For a positive integer k and $T \in L(\mathcal{H})$, T is said to be k-hyponormal if $(I, T, \cdots, T^k)$ is hyponormal. A k-tuple $\mathbf{T} = (T_1, \cdots, T_k)$ is weakly hyponormal if $\lambda_1 T_1 + \cdots + \lambda_k T_k$ is hyponormal for every complex numbers $\lambda_i,\ i = 1, \cdots, k$. An operator T is weakly k-hyponormal if $(T, T^2, \cdots, T^k)$ is weakly hyponormal. The k-hyponormal and weakly k-hyponormal operators play important roles to detect bridges between hyponormal and subnormal operators in $L(\mathcal{H})$. The weak k-hyponormality case in which $k = 2$ has received considerable attention and operators in this class are usually called quadratically hyponormal. In this talk we look into old results on those topics first and discuss recent aspects with our recent results about quadratically hyponormal weighted shifts.

Gregory Knese from Washington University in St. Louis, USA

Title: Canonical Agler decompositions

Abstract: Every Schur function on the bidisk has a natural Hilbert space associated to it analogous to de Branges-Rovnyak spaces on the unit disk. Famous work of Agler shows that this space can be decomposed into two contractively contained Hilbert spaces each of which is invariant under multiplication by one of the coordinate functions. This non-constructive decomposition has remained mysterious for many years, but starting with work of Ball-Sadosky-Vinnikov we have been able to shed much light on these decompositions in recent years. We will discuss the interesting structure that is present in the case of a two variable inner function and explain how this structure generalizes to the case of a non-inner Schur function using scattering systems. This is joint work with Kelly Bickel.

Sasmita Patnaik from IISER Bhopal, India

Title: Subideals of Operators

Abstract: This talk is based on joint work with Gary Weiss. A subideal is an ideal of an ideal of B(H), the algebra of all bounded linear operators on a separable infinite-dimensional complex Hilbert space H. We investigate subideals, a name coined by Weiss and motivated by Fong and Radjavi's 1983 seminal paper on the subject. We determine necessary and sufficient conditions for a subideal generated by sets of cardinality strictly less than the cardinality of the continuum to be also an ideal of B(H). Consequently, we obtain a complete characterization of these subideals.

Santanu Sarkar from IISc, India

Title: The defect sequence for contractive tuples

Abstract: We introduce the defect sequence for a contractive tuple of Hilbert space operators and investigate its properties. The defect sequence is a sequence of numbers, called defect dimensions associated with a contractive tuple. We show that there are upper bounds for the defect dimensions. The tuples for which these upper bounds are obtained, are called maximal contractive tuples. The upper bounds are different in the non-commutative and in the commutative case. We show that the creation operators on the full Fock space and the co-ordinate multipliers on the Drury-Arveson space are maximal. A characterization for a contractive tuple to be maximal is obtained. We give the notion of maximality for a submodule of the Drury-Arveson module on the d-dimensional unit ball B_d. For $d = 1$, it is shown that every submodule of the Hardy module over the unit disc is maximal. But for $d \geq 2$ we prove that any homogeneous submodule or submodule generated by polynomials is not maximal. A characterization of maximal submodules is also obtained.

This is a joint work with T. Bhattacharyya (IISC, Bangalore), B. Krishna Das (ISI, Bangalore) and Jaydeb Sarkar (ISI, Bangalore).

References:

[1] T. Bhattacharyya, B.K. Das, S. Sarkar, The defect sequence for contractive tuples, *Linear Algebra Appl.* 438 (2013), no. 1, 315–330.

[2] B.K. Das, J. Sarkar, S. Sarkar, Maximal contractive tuples, *Complex Analysis and Operator Theory* (2013) (to appear).

[3] H.-L. Gau, P. Y. Wu, Defect indices of powers of a contraction, *Linear Algebra Appl.* 432 (2010), 2824–2833.

Eli Shamovich from Ben Gurion University of the Negev, Israel

Title: Lie Algebra Operator Vessels and General Taylor Joint Spectrum

Abstract: In this talk we will discuss non selfadjoint representations of real finite dimensional Lie algebras. Fixing a basis for the Lie algebra, we can think of the representation as a tuple of operators satisfying certain commutativity conditions. Each such representation we can embed into an operator vessel. The idea of vessels originates in the works of M. S. Livsic and his collaborators (cf. [1]). Essentially a vessel is a representation endowed with additional structure to account for its "non-selfadjointness". We describe the theory of Lie algebra operator vessels and their connection to left-invariant linear systems on the associated simply connected Lie group. We will demonstrate how the theory relates to the general theory of multi-operator spectra developed by J. L. Taylor in his work [2]. We will describe an application for the case of the $ax + b$-algebra.

References: [1] M. S. Livsic, N. Kravitsky, A.S. Markus and V. Vinnikov, *Theory of commuting* *nonselfadjoint operators*, Mathematics and its Applications, Kluwer Academic Publishers Group, Dordrecht, 1995.

[2] J. L. Taylor. *A general framework for a multi-operator functional calculus*, volume 332 of Advances in Mathematics, 9:183-252 (1972).

Vinayak Sholapurkar from S. P. College, Pune, India

Title: Rigidity theorems for spherical hyperexpansions

Abstract: The class of spherical hyperexpansions is a multivariable analog of the class hyperexpansive operators with spherical isometries and spherical 2-isometries being special subclasses. It is known that in dimension one, an invertible 2-hyperexpansion is a unitary. In this talk, we discuss multivariable manifestations of this rigidity theorem. In particular, we provide several conditions on a spherical hyperexpansion which ensure it to be spherical isometry. In the process, we construct several interesting examples of spherical hyperexpansions which are structurally different from the Drury-Arveson m-shift. The work is jointly carried out with Sameer Chavan.

Victor Vinnikov from Ben Gurion University of the Negev, Israel

Title: Vessels of commuting selfadjoint operators

Abstract: An operator vessel, as originally introduced by M.S. Livsic in the 1980s, is a collection of spaces and operators that reflect an interplay between a tuple of operators that commute, or more generally satisfy some commutation relations; it correspond to an overdetermined multidimensional linear input/state/output system together with compatibility conditions for its input and outpout signals. In this talk I will discuss vessels of commuting selfadjoint operators and their functional models on a compact real Riemann surface of dividing type. Such vessels appear naturally in two situations: (a) taking "adjusted" real parts of commuting nonselfadjoint operators satisfying some additional conditions; (b) considering a pair of commuting selfadjoint operators together with an orthogonal decomposition of the space that is "almost" invariant. Case (b) is closely related to developing a generalized dilation theory for certain pairs of commuting nonselfadjoint operators that are ! not dissipative.

This talk is based on joint work with D. Alpay, D. Estevez, and D. Yakubovich.

Kai Wang from Fudan Univesity, China

Title: Reducing subspaces for analytic multipliers of the Bergman space

Abstract: In this talk we will present some recent progress on the structures of the reducing subspaces for the multiplication operator M_ϕ for a finite Blaschke product ϕ on the Bergman space on the unit disk.

Chong Zhao from Fudan Univesity, China

Title: Trace estimation of commutators of multiplication operators on function spaces

Abstract: Let $A = \sum_{k\geq 1} T_{\varphi_k} T^*_{\varphi_k}$ be a bounded linear operator on the Bergman space $L^2_a(B_d)$ or the Hardy space $H^2(B_d)$, where φ_k is a multiplier for each k. We show by trace estimation that for such an operator, the commutators $[A, T_{z_i}]$ belong to the Schatten class $\mathcal{L}^{2p}$ for $p > d$ and $i = 1, \cdots, d$, and satisfy $\|[A, T_{z_i}]\|_{2p} \leq C\|A\|$ for some constant C depending only on p and d. As an application, we find a nearly equivalent condition to the Arveson's conjecture for homogeneous submodules of H^2_d.

General Session

Organizer: **Kaushal Verma**

Piotr Budzy´nski from Department of Applied Mathematics, University of Agriculture in Krakow, Krakow, Poland

Title: On subnormality of unbounded weighted shifts on directed trees

Abstract.The class W of Weighted shifts on directed trees has been introduced quite recently. It contains some classical operators (e.g., the unilateral shift in ‘2) and, what is more interesting, it also contains operators with properties not known before, sometimes very subtle and surprising. In this talk we present an example of a subnormal operator in W whose nth power is densely defined while its (n+1)th power is not. We also show new criteria for subnormality of operators in W, which serve as a basis for constructing our example. We discuss related results concerning subnormality of unbounded weighted composition operators in L2-spaces. This talk is based on a joint work with P. Dymek, Z.J. Jab lo´nski, I.B. Jung and J. Stochel.

J. J. Grobler, North-West University, Potchefstroom, South Africa

Title: Stochastic processes in Riesz spaces: The Kolmogorov-Centsov theorem and Brownian motion

Abstract: We consider continuous time stochastic processe on a Dedekind complete Riesz space on which a conditional expectation is defined. The theorem mentioned in the title is used in the classical theory to prove the existence of a Brownian motion. We prove an abstract version of this theorem and show how this can be used to extend the notion of Brownian motion as defined by Labuschagne and Watson for discrete processes to continuous time stochastic processes.

P. Vinod Kumar from Department of Mathematics, T. M. Govt. College, Tirur, Kerala, India

Title: Minimal and Maximal Operator Space Structures on Banach Spaces

Abstract: A very basic question in operator space theory is to exhibit some particular operator space structures on a given Banach space X. In the most general situation, Blecher and Paulsen [Tensor products of operator spaces., J. Funct. Anal., 99: 262 292, 1991.] achieved this by identifying two extreme operator space structures, Min(X) andMax(X) which represents respectively, the smallest and the largest operator space structures admissible on X In this talk, we consider the subspace and the quotient space struc- ture of minimal and maximal operator spaces. We introduce the notion of hereditarily maximal spaces [Submaximal operator space structures on Ba- nach spaces.,Oper. Matrices, 7(3): 723-732(2013)]. Hereditarily maximal spaces determine a subclass of maximal operator spaces with the property that the operator space structure induced on any subspace coincides with the maximal operator space structure on that subspace. We derive a char- acterization of these spaces and show that the class of hereditarily maximal spaces includes all Hilbertian maximal operator spaces. Also, we prove that if X is a Hilbertan operator space, then any Q -space in X (i.e., any quotient of Min (X)) is minimal. Also, we consider the smallest submaximal space structure on a Banach space X , namely(X), introduced by T. Oikhberg [Subspaces of Maximal Operator Spaces,

Integr.equ.oper.theory, 48 : 81102 , 2004] and derive some characterizations

Surjit Kumar from Department of Mathematics, IIT Kanpur, Kanpur, India

Title: Spherically Balanced Hilbert Spaces of Formal Power Series in Several Variables

Abstract: Consider the complex Hilbert space H2(β) of formal power series in the variables $z_1, \cdot \cdot \cdot, z_m$ endowed with the norm

$$\|f\|^2_{H^2(\beta)} = \int_{\partial\mathbb{B}} \|f_z\|^2_{H^2(\gamma)} d\mu(z) f \in H^2(\beta)$$

where $\beta_0 = 1$. Motivated by the theory of spherical Cauchy dual tuples, we study the spherically balanced spaces, that is, spaces $H^2(\beta)$ for which the multi-sequence $\beta_n \in Z^m_+$ satisfies

$$\|f\|^2_{H^2(\beta)} := \sum_{n \in Z^m_+} |\hat{f}(n)|^2 \beta^2_n (f \in H^2(\beta))$$

for all $n \in Z^m_+$ and $i, j = 1, \cdots, m$,where ϵ_j is the m-tuple with 1 in the jth place and zeros elsewhere. The main result of this paper characterizes spherically balanced spaces: $H^2(\beta)$ is spherically balanced if and only if there exist a Reinhardt measure μ supported on the unit sphere ∂B and a Hilbert space $H^2(\gamma))$ of formal power series in the variable t such that the norm on $H^2(\beta)$ has the slice representation

$$\sum_{k=1}^{m} \frac{\beta^2_{n+\epsilon_i+\epsilon_k}}{\beta^2_{n+\epsilon_i}} = \sum_{k=1}^{m} \frac{\beta^2_{n+\epsilon_j+\epsilon_k}}{\beta^2_{n+\epsilon_j}}$$

where $f_z(t) = f(tz)$ is a formal power series in the variable t. In particular, the pair $[\mu, H^2(\gamma)]$ provides a complete invariant for spherically balanced spaces.

M. N. N. Namboodiri from Department of Mathematics, "CUSAT" - Cochin, India

Title: Korovkin-type theorems via completely positive/bounded maps on operator algebras-recent developments

Abstract:The classical as well as the noncommutative Korovkin-type theorems deal with the convergence of positive linear maps, with respect to different modes of convergence, like norm or weak operator convergence etc. [5]. In this lecture, apart from presenting recent developments in this area that includes new versions of Korovkin-type theorems under the notions of convergence induced by strong, weak and uniform eigenvalue clustering of matrix sequences with growing order [4],the connection between operator systems/spaces,Korovkin sets and problems related to the associated spectral condions are considered.

References: [1] F. Altomare, M. Campiti, Korovkin type approximation theory and its applications, de Gruyter Studies in Mathematics, Berlin, New York, 1994.

[2] W. B. Arveson, Subalgebras of C*-algebras, Acta. Math. 123 (1969), 141– 224.

[3] W. B. Arveson, Subalgebras of C*-algebras II, Acta. Math. 128 (1972), 271–308.

[4] K. Kumar, M.N.N. Namboodiri, S.Serra-Capizano,Preconditioners and Korovkin-type Theorems for infinite dimensional bounded linear operators via Completely Positive Maps, Studia Mathematica(2) 2013-In print

[5] M. N. N. Namboodiri, Developments in noncommutative Korovkin-type theorems, RIMS Kokyuroku Bessatsu Series [ISSN1880-2818] 1737-Non Commutative Structure Operator Theory and its Applications, 2011

[6] M. Uchiyama, Korovkin type theorems for Schwartz maps and operator monotone functions in C*-algebras, Math. Z. 230, 1999.

Vladimir Peller, Michigan State University, USA

Title:Estimates for Lipschitz functions of perturbed self-adjoint operators based on finite dimensional estimates.

Abstract: I am going to speak about my joint results with Aleksandrov. We offer an approach to estimate the norms $\|f(A) - f(B)\|$ for Lipschitz functions f and self-adjoint operators A and B that is based on finite-dimensional estimates.

Lucijan Plevnik from Institute of Mathematics, Physics, and Mechanics, Ljubljana, Slovenia

Title: Maps preserving complementarity of closed subspaces of a Hilbert space

Abstract: Let H and K be infinite-dimensional separable Hilbert spaces and LatH the lattice of all closed subspaces of H. We describe the general form of pairs of bijective maps Φ,Ψ : LatH $\rightarrow$ LatK having the property that for every pair U, V $\in$ LatH we have H = U $\oplus$ V () K = Φ(U) $\oplus$ Ψ(V). Then we reformulate this theorem as a description of bijective image equality and kernel equality preserving maps acting on bounded linear idempotent operators. Several known structural results for maps on idempotents are easy consequences. This is joint work with Peter Semrl.

Peter Semrl from Faculty of Mathematics and Physics, University of Ljubljana, Ljubljana, Slovenia

Title: Adjacency preserving maps

Abstract: I will present several recent results on adjacency preserving maps on matrices and operators. Applications in mathematical physics will be discussed.

References: [1] Peter ˇSemrl: Comparability preserving maps on Hilbert space effect algebras, Comm. Math. Phys. 313 (2012), 375-384.

[2] Peter ˇSemrl: Symmetries on bounded observables - a unified approach based on adjacency preserving maps, Integral Equations Operator Theory 72 (2012), 7-66.

[3] Peter ˇSemrl: The optimal version of Hua's fundamental theorem of geometry of rectangular matrices, accepted in Mem. Amer. Math. Soc.

[4] Peter ˇSemrl: Symmetries of Hilbert space effect algebras, accepted in J. Lond. Math. Soc.

[5] Peter ˇSemrl: Hua's fundamental theorem of geometry of rectangular matrices over EAS division rings, preprint.

Prahlad Vaidyanathan from Department of Mathematics, IISER Bhopal, Bhopal, India

Title: E-theory for Continuous Fields of C Algebras*

Abstract: A continuous field of C* algebras is a family of C* algebras parametrized over a locally compact space, and can be thought of as a generalization of the space of continuous sections of a vector bundle. These objects are of importance because every non-simple C* algebra with a Hausdorff

spectrum is a continuous field over its primitive ideal space. In this talk, we give an introduction to the problem of classifying these objects and understanding homomorphisms between them. We explain the role of the equivariant E-theory group, and present some recent results that compute this group for a class of continuous fields over the unit interval. As a result, we show that algebras in this class are completely classified by an ideal-related K-theoretic invariant.

Martin Weigt from Department of Mathematics and Applied Mathematics, Nelson Mandela Metropolitan University, Port Elizabeth, South Africa.

Title: Unbounded derivations of commutative generalized B-algebras*

Abstract: Generalized B-algebras(GB-algebras for short) are locally convex topological*-algebras which are generalizations of C*-algebras, and were first studied by G.R. Allan in [1], and later by P. G. Dixon in [3]. These algebras are also abstract unbounded operator algebras, which are important in mathematical physics. Besides C*-algebras, examples of GB*-algebras include inverse limits of C*- algebras, called pro-C*-algebras, and non-commutative Arens algebras. It is well known that the zero derivation is the only derivation of a commutative C*-algebra, and that every derivation of a C*-algebra is continuous. These results have been extended to commutative pro- C*-algebras in [2], and to commutative Fr´echet GB*-algebras in [4]. The theory of unbounded derivations of C*-algebras and their applications to mathematical physics is well developed, and in this talk, we give some results about unbounded derivations of commutative GB*-algebras. In obtaining these results, we borrow some techniques from commutative algebra which seem not to be used in functional analysis. We also give an example of a commutative GB*-algebra having a nonzero (everywhere defined) derivation. The results presented in this talk is joint work with I. Zarakas (Department of Mathematics, University of Athens, Greece).

References: [1] G. R. Allan, On a class of locally convex algebras, Proc. London Math. Soc. 17, 1967, 91–114.

[2] R. Becker, Derivations on LMC*-Algebras, Math. Nachr., 155, 1992, 141– 149.

[3] P.G. Dixon, Generalized B-algebras, Proc. London Math. Soc. 21(1970), 693-715.

[4] M. Weigt and I. Zarakas, Derivations of Generalized B-algebras, to appear in Extracta Mathematicae.

Janusz Wysoczanski from Institute of Mathematics, Wroclaw University, Poland

Title: On generalized anyon statistics

Abstract: We shall present a construction of generalized Q-symmetrization operator, which allows to define Q-creation/annihilation operatorson a deformed Fock space. These satisfy (pointwise) generalized Qcommutation relations Moreover, the notion of Q-cumulants and Qindependence will be introduced. This is based on joint work with E. Lytvynov and M. Bozejko, published in Commun. Math. Phys. 2012.

Spectral theory and differential operators

Organizers: **Paul Binding, Tom ter Elst and Carsten Trunk**

Rostyslav Hryniv (IAPMM, Lviv, Ukraine and Rzeszów University, Poland

Title: Reconstruction of Sturm–Liouville operators with energy-dependent potentials

Abstract: We study the direct and inverse spectral problems for energy-dependent Sturm–Liouville equations arising in many models of classical and quantum mechanics. In contrast to the classical case, energy-dependent Sturm-Lioville problems with real-valued potentials can possess nonreal and nonsimple spectra. We give a complete characterization of their spectra and suitably defined norming constants and then solve the inverse problem of reconstructing energy-dependent Sturm–Liouville equations from either two spectra or one spectrum and the sequence of the norming constants. The approach is based on connection between the spectral problems under consideration and those for Dirac operators of special form.
The talk is based on a joint project with Natalia Pronska (Lviv, Ukraine).

Andreas Ioannidis, Linnæus University, Sweden

Title: The eigenvalue problem for the cavity Maxwell operator

Abstract: In this talk we discuss the propagation problem of the time harmonic electromagnetic field $\mathsf{e} := (\mathbf{E}, \mathbf{H})^T$ (time convention is taken to be $\mathrm{e}^{-\mathrm{i}\omega t}$) inside a Lipschitz bounded domain $\Omega \subset \mathbb{R}^3$. Ω represents a cavity, which is filled with a general bianisotropic medium and is perfect conducting, that is, the boundary condition $\hat{\mathbf{n}} \times \mathbf{E} = \mathbf{0}$ on $\partial\Omega$ is satisfied. $\hat{\mathbf{n}}$ denotes the outward normal, which is defined almost everywhere on $\partial\Omega$.
This problem is formally formulated as an eigenvalue problem for a linear operator pencil

$$\mathcal{Q}\mathsf{e} = \omega \mathsf{M}(\omega)\mathsf{e}. \tag{$\star$}$$

Here $\mathcal{Q}$ stands for the self-adjoint Maxwell operator

$$\mathcal{Q} := \mathrm{i}\begin{bmatrix} 0 & -\operatorname{curl} \\ \operatorname{curl} & 0 \end{bmatrix},$$

and $\mathsf{M} = \mathsf{M}(\omega)$ for the material matrix

$$\mathsf{M} := \begin{bmatrix} \boldsymbol{\varepsilon} & \boldsymbol{\xi} \\ \boldsymbol{\zeta} & \boldsymbol{\mu} \end{bmatrix},$$

which models the medium properties inside Ω. The entries of M are $L^\infty(\Omega)$ functions that depend on the eigenvalue ω.
We realize ($\star$) in a Hilbert space setting and our analysis breaks down into two steps:

1. We first study the hollow cavity, modeled by the simplest case $\mathsf{M} = I$. Then ($\star$) becomes a standard eigenvalue problem for which we employ a variant of the spectral theorem tfor discrete self-adjoint operators to prove existence for the eigenelements.

2. For the general matrix operator M, we use perturbation arguments to prove existence for the eigenelements of the general cavity and characterize them with aim of their hollow cavity counterparts.

Vadim Kostrykin, Institut für Mathematik, Johannes Gutenberg-Universität Mainz,Germany

Title: The div A grad without ellipticity

Abstract: The talk discusses some recent results on div A grad-operators for sign-indefinite coefficient matrices A. A simplest example of such kind is $\mathcal{L} = -\frac{d}{dx}\mathrm{sign}(x)\frac{d}{dx}$ on a bounded interval. Using the representation theorem for indefinite quadratic forms, for a wide class of coefficient matrices we prove the existence of a unique self-adjoint, boundedly invertible operator $\mathcal{L}$, associated with the form $\langle$grad u, Agrad u$\rangle$.

Maria O.Kovaleva, St. Petersburg National Research University of Information Technologies, Mechanics and Optics, St. Petersburg, Russia

Title: Stokes graph and non-oscillating solutions

Abstract: Stokes flows with varying viscosity η and density ρ are considered. We deal with the case when the flow is concentrated in narrow neighborhood of a network. The main term of the asymptotic expansion (in respect to the width of these narrow channels) for the flow velocity satisfies the 1D Schrödinger equation with a specific potential:

$$v'' - \frac{\eta'}{\eta}\frac{\rho'}{\rho}v = 0.$$

Here $v = v(x)$ is the flow velocity. It allow one to use the metric graph with the Schödinger operator $L = -\frac{d^2}{dx^2} + \frac{\eta'}{\eta}\frac{\rho'}{\rho}$ on edges for the description of the flow. We call it the Stokes graph. It is analogous to the corresponding quantum graph. It is necessary to determine boundary conditions at the graph vertices. Consider a vertex (let it be zero point) with n output edges. From physical conditions one has $\rho_1(0) = \rho_2(0) = ...\rho_n(0) = \rho(0)$ and $v_1'(+0) = v_2'(+0) = ...v_n'(+0) = v'(0)$. The last condition is related with the pressure continuity. Here $v_j'(+0)$ is the derivative in the outgoing direction at the vertex 0. The continuity equation gives us for this vertex:

$$\sum_{j=1}^{n} v_j = -\left(\frac{\rho(0)}{\sum\limits_{j=1}^{n}\rho_j'(+0)}\right)v'(0).$$

It is analogous to well-known δ'-coupling condition for quantum graph. The coupling constant is related with the density derivative.

Non-oscillating solutions on the Stokes graph are studied. Estimates analogous to the Harnack's inequality for an elliptic operator on a manifold are obtained. The relation between the spectral properties of the graph Hamiltonian and the parameters of the corresponding Stokes flow is discussed.

Kiran Kumar, Department of Mathematics, University of Calicut - Kerala (India)

Title: Truncation method for random bounded self-adjoint operators

Abstract: In this talk, I wish to discuss the linear algebraic techniques for approximating the spectrum of bounded random self-adjoint operators on separable Hilbert spaces. The random version of the truncation method used to approximate spectrum of a bounded self-adjoint operator in [1], is presented here. The Wigner operators are considered here as their truncations will become the well known Wigner matrices, and the eigenvalue distributions of such random matrices of large order are used in the spectral approximation problem. The infinite dimensional operator version of the estimations by Tao and Vu in [7] is done. Finally, a new method - analogous to the quadratic projection method and second order relative spectra, used in [2, 3, 5, 6] - is proposed to predict the spectral gaps that may arise between the bounds of essential spectrum of a bounded self-adjoint operator. This talk is based on the recent work [4]
[1] A. Böttcher, A.V. Chithra, M.N.N. Namboodiri; *Approximation of Approximation Numbers by Truncation* J. Integr. Equ. Oper. Theory 2001,39, 387-395.
[2] E.B. Davies, *Spectral Enclosures and complex resonances for self-adjoint Operators*, LMS J. Comput. Math.1 (1998), 42–74.2
[3] E.B. Davies, M. Plum, *Spectral Pollution*, IMA Journal of Numerical Analysis 24(2004), 417–438.
[4] K. Kumar *Truncation Method For Random Bounded Self-Adjoint Operators*(Communicated).
[5] L. Boulton, M. Levitin; *On Approximation of the Eigenvalues of Perturbed Periodic Schr¨odinger Operators*, arxiv;math/0702420v1 (2007).
[6] M. Levitin, E. Shargorodsky *Spectral pollution and second-order relative spectra for self-adjoint operators*, IMA Journal of Numerical Analysis 24(2004), 393–416.
[7] T. Tao, V. Vu ; *Random Matrices: Localization of the eigenvalues and the necessity of four moments* Acta Mathematica Vietnamica,2011 Volume 36, Number 2, 431-449.

A. I. Popov, St. Petersburg National Research University of Information Technologies, Mechanics and Optics, St. Petersburg, 197101, Russia
Title: Spectral analysis for three coupled strips quantum graph
Abstract: Infinite quantum graph Γ formed by three coupled identical strips with honeycomb lattice of edges E and vertices V is considered. The Schrödinger operator is constructed in the framework of the theory of self-adjoint extensions of symmetric operators in the Hilbert spaces. It is defined at each edge by the following differential expression

$$H = -\frac{d^2}{dx^2}. \tag{1}$$

The domain of H is as follows (we assume δ-coupling at the vertices):

$$\begin{cases} f \in W_2^1(\Gamma) \bigcap W_2^2(\Gamma \setminus V), \\ \sum_{e \in E_v} \frac{df}{dx_e}(v) = \alpha f(v). \end{cases} \tag{2}$$

Here α is some fixed number, $E_v := \{e \in E \,|v \in e\}$ is the set of edges adjacent to the vertex $v, v \in V$. The sum is taken over all edges e incident to the vertex v and the derivatives are taken along e in the directions away from the vertex v (outgoing direction).
The spectral analysis of the Hamiltonian is made for different values of the parameter α. The method of the transfer-matrices is used. The spectral equation is obtained in an explicit form, The essential spectrum has band structure. The condition on the operator parameter α ensuring the existence of

eigenvalues in the gaps is obtained. For positive α the operator is positive. For negative α the negative lower bound of the essential spectrum is found. We specify the restrictions on the model parameters ensuring the existence of the eigenvalue below the threshold.

Pierre Portal, Australian National University, Mathematical Sciences Institute, Australia
Title: Non-autonomous parabolic systems with rough coefficients
Abstract: We consider problems of the form

$$\begin{cases} \partial_t u(t,x) = div(A(t,.)\nabla u)(t,x) \quad t \geq 0 \quad x \in \mathbb{R}^n, \\ u(0,.) = f \in L^p(\mathbb{R}^n). \end{cases}$$

for uniformly elliptic coefficients $A(t,.) \in L^\infty(\mathbb{R}^n; M_n(\mathbb{C}))$. In the real valued case, the theory of such problems is well established, and no regularity in the time variable is required. The methods, however, break down in the complex valued case (or more generally for systems). The case of systems can be treated by abstract operator theoretic methods, but these methods impose some time regularity (continuity at least). Here we present a new approach that allows us to treat problems with coefficients which are of bounded variation in time. This is based on results about maximal regularity operators on tent spaces, and is joint work with Pascal Auscher and Sylvie Monniaux.

Jonathan Rohleder, Graz University of Technology
Title: Titchmarsh–Weyl theory for elliptic differential operators
Abstract: In this talk elliptic differential operators of the form

$$\mathcal{L} = -\sum_{j,k=1}^{n} \partial_j a_{jk} \partial_k + \sum_{j=1}^{n} \left(a_j \partial_j - \partial_j \overline{a_j} \right) + a$$

with variable coefficients on $\mathbb{R}^n$ and on Lipschitz domains $\Omega \subset \mathbb{R}^n$ are considered. We discuss spectral properties of selfadjoint realizations of $\mathcal{L}$ and its connection to corresponding Titchmarsh–Weyl functions / Dirichlet-to-Neumann maps.

Alexei Rybkin, Department of Mathematics and Statistics, University of Alaska Fairbanks, Fairbanks
Title: On the Hankel operator approach to completely integrable systems
Abstract: Completely integrable systems and the theory of Hankel/Toeplitz operators are very large and active theories that have remained essentially hermetic to each other. In this talk we demonstrate that there are some deep links between the two. On the prototypical example of the Cauchy problem for the Korteweg-de Vries (KdV) equation we demonstrate the power of the language of Hankel operators in which symbols are conveniently represented in terms of the scattering data for the Schrodinger operator associated with the initial data for the KdV equation. This approach recovers and improves on many already known results as well as yields to a variety of new ones. The main new result is the well-posedness of the Cauchy problem for the KdV equation with initial data behaving essentially arbitrary at minus infinity and decay sufficiently fast at plus infinity.
The talk is based on joint work with Sergei Grudsky.

Petr Siegl, Mathematical Institute, University of Bern, Switzerland
Title: Root system of perturbations of harmonic and anharmonic oscillators
Abstract: We analyze perturbations of the harmonic oscillator type operator in a Hilbert space, i.e. of the self-adjoint operator with simple positive eigenvalues μ_k satisfying $\mu_{k+1} - \mu_k \geq \Delta > 0$. Perturbations are considered in the sense of quadratic forms. Under a "local subordination assumption", the eigenvalues of the perturbed non-self-adjoint operator become eventually simple and the root system forms a Riesz basis.
The abstract results are applied to harmonic and anharmonic oscillators and perturbations by singular potentials are particularly considered. Finally, we present classes of perturbations for which the eigensystem of the perturbed oscillators is not even a basis; the norms of spectral projections are found to grow at rates from arbitrarily slowly to exponentially rapidly.
The talk is based on:
[1] B. Mityagin, P. Siegl: *Root system of singular perturbations of the harmonic oscillator type operators*, arXiv:1307.6245,
[2] B. Mityagin, P. Siegl, J. Viola: *Differential operators admitting various rates of spectral projection growth*, arXiv:1309.3751.

Carsten Trunk, TU Ilmenau, Germany
Title: On a class of Sturm-Liouville operators which are connected to $\mathcal{PT}$ quantum mechanics
Abstract: We consider so-called $\mathcal{PT}$ symmetric operators in the space $(L_2(\mathbb{R}), [\cdot,\cdot])$, with an indefinite inner product $[\cdot,\cdot]$ given via the fundamental symmetry $\mathcal{P}f(x) = f(-x)$ such that $[f, g] = (\mathcal{P}f, g)_{L_2}$. The space $(L_2(\mathbb{R}), [\cdot,\cdot])$ is an example of a Krein space. The action of the anti-linear operator $\mathcal{T}$ on a function of a real spatial variable x is defined by $\mathcal{T}f(x) = \overline{f(x)}$.
An operator A is said to be $\mathcal{PT}$-symmetric if it commutes with $\mathcal{PT}$.
In the last decade a generalization of the harmonic oscillator using a complex deformation was investigated. This operator is defined via the differential expression

$$(\tau y)(x) := -y''(x) + x^2(ix)^\epsilon y(x), \quad \epsilon > 0.$$

We will start our investigations with the discussion of some simple cases (like ϵ even) and we will concentrate on a describtion of self-adjoint, $\mathcal{P}$-selfadjoint and $\mathcal{PT}$-symmetric operators related to such a differential expression and their spectral properties. The talk is based on joint works with T.Ya. Azizov (Voronezh).