MATRICES AND OPERATORS
CONFERENCE IN HONOUR OF SIXTIETH BIRTHDAY OF RAJENDRA BHATIA
DATES: 27 TO 30 DECEMBER, 2012
VENUE: INDIAN INSTITUTE OF SCIENCE.
TITLES AND ABSTRACTS

Speaker: Tsuyoshi Ando, Hokkaido University.

Title: Central Decomposition of Operators.

Abstract: Let $\mathscr{H}$ be a Hilbert space and $B(\mathscr{H})$ the space of bounded linear operators on $\mathscr{H}$. For $T \in B(\mathscr{H})$ with polar decomposition $T = U \cdot |T|$, A. Aluthge introduced the (so-called) *Aluthge transform* $\tilde{T} := |T|^{\frac{1}{2}} \cdot U \cdot |T|^{\frac{1}{2}}$. It has been recognized that the Aluthge transform is quite useful in the study of structures of operators of some kinds.

In this talk I want to go in the reverse direction.But I will treat only a *quasi-invertible* operator $T \in B(\mathscr{H})$ in the sense $\ker(T) = \{0\}$ and $\ker(T^*) = \{0\}$.

A quasi-invertible operator T will be said to be *Aluthge decomposable* if it admits an *Aluthge decomposition* in the sense

$$T = AVA \quad \text{with unitary } V \text{ and injective } A \geq 0.$$

More generally T will be said to be *centrally decomposable* if it admits a *central decomposition* in the sense

$$T = AVA^* \quad \text{with unitary } V \text{ and quasi-invertible } A.$$

Here the unitary operator V will be called the *argument operator* of the decomposition while AA^* is the *weight operator*.
Contrary to the case of polar decomposition, Aluthge decomposition does not always exist, and is not always unique even when it exists. We will say that *stability of inertia* holds for a centrally decomposable operator T if argumement operators in any two of its central decompositions are (unitarily) similar.

Matrix theorists have been discussing central decomposition and stability of inertia for matrices in order to generalize the Sylvester law of inertia for Hermitian matrices to general matrices.

In this talk I will consider the following problems.

(i) *Which quasi-invertible operator is centrally decomposable ?*

(ii) *For which centrally decomposable operator does the stability of inertia hold ?*

(iii) *How can Aluthge decompositions be constructed for a matrix ?*

(iv) *When does a matrix admit a unique Aluthge decomposition ?*

Speaker Dario Bini, Universita' di Pisa.

Title: Structured matrix geometric means: theory and algorithms.

Abstract: (joint work with B. Iannazzo, B. Jeuris, R. Vandebril) The geometric mean of a set of positive definite matrices $A_1, \ldots, A_p$ is usually identified with the Karcher mean G which verifies most of the desirable properties of the scalar geometric mean. Unfortunately, the Karcher mean does not generally preserve the structure of the input matrices. Say, if A_i, $i = 1, \ldots, p$ are Toeplitz, then G is not Toeplitz in general.
In this talk we introduce a definition of geometric mean which preserves the structure of the input matrices, satisfies most of the Ando-Li-Mathias properties and is easily computable. This mean is expressed in terms of the solution of a vector equation. We introduce some algorithms for the numerical solution of this equation and provide a complete analysis of their convergence speed.
Numerical experiments performed with Toeplitz matrices confirm the effectiveness of our approach.

Speaker: Man-Duen Choi, University of Toronto.

Title: The Taming of the Shrew.

Abstract: I wish to tame the physical quantum entanglements (in disguise of non-commutative geometry), by means of pure mathematics. Note that the research work along these lines, has been proven to be useful to the foundation of abstract quantum information in the light of (the reality of) quantum computers.

Speaker: Chandler Davis, University of Toronto.

Title: A Representation Theorem For Operators.

Abstract: The object represented by the theorem is an arbitrary linear operator from one Hilbert space to another, togather with a distinguished subspace. This is applied to give a general theory of certain moment problems.

Speaker: Pedro J Freitas, University of Lisbon.

Title: Derivatives of Functions of Matrices.

Abstract: There have been recent results concerning directional derivatives, of degree higher than one, for the determinant, the permanent, the symmetric and the antisymmetric power maps, by R. Bhatia, P. Grover and T. Jain. We present generalizations of these results for immanants and other symmetric powers.

Speaker: Shmuel Friedland, University of Illinois at Chicago.

Title: The global and local additivity problems in
quantum information theory.

Abstract: The capacity of the classical channel was investigated by Claude Shannon in 1948. This capacity is additive under the tensor products of two channels. The capacity of a quantum channel (QC) was introduced by Alexander Holevo in 1998 [3]. One of the fundamental problems in quantum information theory is whether the capacity of QC is additive or not under the tensor product. Equivalently, can entangled input increase the quantum capacity? In 2009 Matthew Hastings [2] gave a positive answer to this problem! It was shown by Peter Shor [4] in 2004 that the additivity of the Holevo capacity is equivalent to:

(i) additivity of the minimum entropy output of a quantum channel,

(ii) additivity of the entanglement of formation,

(iii) strong superadditivity of the entanglement of formation.

Contrary to the above result we will show that the *local* minimum entropy output of a quantum channel is additive. This result is a joint work with Gilad Gour [1].

References:
[1] G. Gour and S. Friedland, The minimum entropy output of a quantum channel is locally additive, arXiv:1105.6122.
[2] M. B. Hastings, Nature Physics **5**, 255 (2009).
[3] A. S. Holevo, The additivity problem in quantum information theory. In *International Congress of Mathematicians. Vol. III*, pages 999–1018. Eur. Math. Soc., Zürich, 2006.
[4] P. W. Shor, Equivalence of additivity questions in quantum information theory. *Comm. Math. Phys.*, 246(3):453–472, 2004.

Speaker: Fumio Hiai, Tohoku University.

Title: Concavity of certain matrix trace and norm functions.

Abstract: Lieb's concavity/convexity and Epstein's concavity are extended by improving Epstein's method to matrix trace functions in certain general forms, which are further generalized by the majorization method to concavity/convexity of similar functions under symmetric (anti-) norms in place of the trace.

Speaker: II Bong Jung, Kyungpook National University.

Title: A Non- Hypernormal Operator Generating Steltjes Moment Sequences.

Abstract: A linear operator S in a complex Hilbert space $\mathscr{H}$ for which the set $\mathscr{D}^\infty(S)$ of its C^∞-vectors is dense in $\mathscr{H}$ and $\{\|S^n f\|^2\}_{n=0}^\infty$ is a Stieltjes moment sequence for every $f \in \mathscr{D}^\infty(S)$ is said to generate Stieltjes moment sequences. It is shown that there exists a closed non-hyponormal operator S which generates Stieltjes moment sequences. What is more, $\mathscr{D}^\infty(S)$ is a core of any power S^n of S. This is established with the help of a weighted shift on a directed tree with one branching vertex. The main tool in the construction comes from the theory of indeterminate Stieltjes moment sequences. As a consequence, it is shown that there exists a non-hyponormal composition operator in an L^2-space over a σ-finite measure space which is injective, paranormal and which generates Stieltjes moment sequences. (This is a joint work with Z. Jablonski and J. Stochel.)

Speaker: Rien Kaashoek, VU University,Amsterdam.

Title: Szegő-Kreĭn orthogonal matrix functions on the unit circle or the real line: inverse problems and band method.

Abstract: In these lectures inverse problems for Szegő-Kreĭn orthogonal matrix functions will be put into an abstract algebraic setting which is known as the band method. The abstract approach allows one to reduce the inverse problems to linear matrix function equations. The latter equations involve matrix polynomials in the discrete case, and entire matrix functions in the continuous case. For the entire matrix functions we shall describe in detail the various elements that are used in solving the equations. The paper mentioned below serves as our main source. The plan of the lectures is as follows:

(i) Lecture 1: Szegő-Kreĭn polynomials and Kreĭn entire functions – an introduction.

(ii) Lecture 2: The band method.

(iii) Lecture 3: Entire matrix function equations.

References:

[1] M.A. Kaashoek and L. Lerer, The band method and inverse problems for orthogonal matrix functions of Szegő-Kreĭn type, *Indagationes Math.* **23** (2012), 900–920.

Speaker: Rajeeva Karandikar, Chennai Mathematical Institute.

Title: The matrix geometric mean.

Abstract: An attractive candidate for the geometric mean of several positive definite matrices is their Riemannian barrycentre. We give a simple probabilistic proof of one of its important properties, monotonicity in its arguments. This had been established earlier by J. Lawson and Y. Lim. This is joint work with Rajendra Bhatia. I will also mention an open problem (unrelated to geometric mean, but close to Rajendra's interests).

Speaker: Sangho Kum, Chungbuk National University.

Title: No Dice Theorem on Symmetric Cones.

Abstract: In this talk, we study the Karcher mean on symmetric cones, self-dual homogeneous convex cones, which are Riemannian symmetric spaces of non-compact type. A deterministic approximation to the Karcher mean is presented via an extension of Holbrook's no dice theorem for the conves cone of positive definite matrices.

Speaker: Ren-Cang Li, University of Texax at Arlington.

Title: Classical Min-max Principles and Beyond.

Abstract: The classical min-max principles of Courant, Fisher, Lidskii, and Wielandt for the Hermitian eigenvalue problem $Ax = \lambda x$ were developed over a span of about five decades from the 1900s to 1950s. These principles are very fundamental in the theory of eigenvalues, and contribute in many ways to our deep understanding of the problem as well as today's efficient numerical methods for solving small and large scale Hermitian eigenvalue problems. Owing to such prominent importance, extensions of these min-max principles are of great theoretical and practical interest for eigenvalue problems beyond the Hermitian one. In this talk, we will survey some of these extensions for generalized Hermitian eigenvalue problem $Ax = \lambda Bx$ with indefinite and possibly singular B, quadratic eigenvalue problem $(A\lambda^2 + B\lambda + C)x = 0$, and linear response eigenvalue problem arising from time-dependent density functional theory (TDDFT). We will also explain their potential numerical implications.
This is a joint work with Zhaojun Bai (University of California at Davis) and Xin Liang (Peking University).

Speaker: Yongdo Lim, Kyungpook National University.

Title: Karcher means and Karcher equations of positive definite matrices.

Abstract: The Karcher mean (Cartan mean, Riemannian center of mass, least squares mean or Riemannian geometric mean) of positive definite matrices has sprung up as the most attractive averaging among other matrix geometric means.
The fact that the Karcher mean is the unique positive definite solution of the Karcher equation is crucial for deriving its properties and for various numerical methods of computing the Karcher mean. In this talk we present an elementary proof by using only basic matrix theory together with a minimal knowledge of Riemannian geometry and then we clarify the relationship between the least-squares mean and the critical point of the objective function in both the Euclidean and the Riemannian sense.

Speaker: J.Martin Lindsay, Lancaster University.

Title: Elementary Evolutions In Banach Algebra.

Abstract: As is well known, the continuous one-parameter semigroups in a unital Banach algebra $\mathscr{A}$ are precisely the families

$$\{(e^{ta})_{t\geq 0} : a \in \mathscr{A}\}.$$

The classical proof of this fact does not extend to anything analogous for evolutions. By an *evolution* in $\mathscr{A}$ we understand a family $(e_{r,t})_{0\leq r\leq t}$ in $\mathscr{A}$, satisfying $e_{r,t} = 1$ for all $r \geq 0$ and $e_{r,s}e_{s,t} = e_{r,t}$ for all $0 \leq r \leq s \leq t$; we call it *continuous* if it is continuous in each argument. It is not hard to see that the continuous evolutions in $\mathscr{A}$ are precisely the families

$$\{(\phi_r^{-1}\phi_t)_{0\leq r\leq t} : \phi \in C(\mathbb{R}_+; \mathscr{A}^\times)\},$$

where $\mathscr{A}^\times$ denotes the group of units of $\mathscr{A}$; in particular, as for semigroups, continuity implies invertibility.

In this talk an elementary subclass of continuous evolutions (which have well-defined 'generators') is identified and some of its properties are explored. In recognition of Rajendra's service to the exposition of mathematics, the talk is designed to be accessible to PhD students.

This is joint work with B. Krishna Das. It is supported by the UKIERI Research Collaboration Network *Quantum Probability - Noncommutative Geometry - Quantum Information*

Speaker: Nobuaki Obata, Tohoku University.

Title: Asymptotic Spectral Distributions of Distance k-Graphs of Cartesian Product Graphs.

Abstract: Quantum probability theory provides a framework of extending the measure-theoretical (Kolmogorovian) probability theory. The idea traces back to von Neumann (1932), who, aiming at the mathematical foundation for the statistical questions in quantum mechanics, initiated a parallel theory by making a selfadjoint operator and a trace play the roles of a random variable and a probability measure, respectively. During the last 30 years quantum probability theory has developed considerably with wide applications. Among others, our interest lies in asymptotic spectral analysis of growing graphs [1,2].

In this lecture, following [3], we report a new type of limit distributions obtained from the distance k-graph of the N-fold Cartesian power of a graph G as $N \to \infty$. More precisely, let G be a finite connected graph on two or more vertices and $G^{[N,k]}$ the distance k-graph of the N-fold Cartesian power of G. For a fixed $k \geq 1$, we obtain explicitly the large N limit of the spectral distribution (the eigenvalue distribution of the adjacency matrix) of $G^{[N,k]}$. The limit distribution is described in terms of the Hermite polynomials.

References:

[1] A. Hora and N. Obata: Quantum Probability and Spectral Analysis of Graphs, Springer, 2007.

[2] N. Obata: Spectral Analysis of Large Networks:Quantum Probabilistic Approach and Applications, Lectures delivered at Chungbuk National University, March-May, 2010. Downloadable at www.math.is.tohoku.ac.jp/~obata

[3] Y. Hibino, H. H. Lee and N. Obata: Asymptotic Spectral Distributions of Distance k-Graphs of

Cartesian Product Graphs, preprint, 2012.

Speaker: Abbas Salemi Parizi, Shahid Bahonar University of Kerman.

Title: Polynomial Numerical Hulls of Matrices and GMRES Method.

Abstract: This is an exposition of some aspects of polynomial numrical hulls of matrices. Also, we study the problem of complete and partial stagnation of the generalized minimum residual (GMRES) method.

Speaker: K. R. Parthasarathy, Indian Statistical Institute Delhi.

Title: Positivity,Probability and Statistics.

Abstract: This will be an expository talk intended as a guided tour through the flower garden of positivity commenting on gaussian distributions,GNS principle and group actions, infinite divisibility, quantum Fourier transform and gaussian states and extremal elements of some convex subsets in the space of positive matrices.

Speaker: Takashi Sano, Yamagata University.

Title: Loewner and Kwong matrices.

Abstract: I would like to review results on Loewner and Kwong matrices, including joint work with Professors Bhatia, Hiai, and recent one with graduate students: Hidaka and Tachibana.

Speaker: Peter Semrl, University of Ljubljana.

Title: Adjacency preserving maps.

Abstract: Two matrices are said to be adjacent if one is a rank one perturbation of the other one. Hua's theorems of geometry of matrices characterize bijective maps on various spaces of matrices preserving adjacency in both directions. We will present several improvements of these results as well as applications in matrix theory, geometry and mathematical physics.

Speaker: Stefano Serra, University of Insubria.

Title: Symbol approach (for structured matrices) in multigrid and preconditioning.

Abstract: We review spectral properties and solution methods of multigrid type or of preconditioned Krylov type for linear system of large dimensions and structured type, arising from the approximation of integro-differential operators.

Speaker: Dinesh Singh, University of Delhi.

Title: Multiplication by Monomials on BMOA.

Abstract: In a recent paper, the author (with Davidson et al) sparked off an area in-what has now become standard parlance- 'Constrained Interpolation' of the Nevanlinna-Pick type. The main thrust in constrained interpolation is to seek necessary and sufficient conditions for interpolation by sub-algebras of Banach algebra of bounded holomorhic functions. on the disk. The solution of the problem is closely dependent on identifying invariant subspaces for these subalgebras. This talk deals with looking at subspaces invariant under some of these linear transformations for the Banach space of holomorphic functions of bounded mean oscillation. The challenge is to overcome the difficulties of the BMOA norm. An interesting application of the techniques is a very short proof of a well known and deep result on the operator S^* acting on the Hardy space H^1. This is a report on joint work with N. Sahni.

Speaker: Kalyan Bidhan Sinha, Jawaharlal Nehru Centre for Advanced Scientific Research.

Title: Trace Mean-value formulae for for Functions of one and two self-adjoint operators .

Abstract: Beginning with Krein , trace formulae for differences of functions of operators under perturbations have been derived . Koplienko extended to the second order , though his proof was incomplete . More recently , Dykema and Simon et al had a fresh look at it . The present author , with Arup Chattopadhyay , gave a proof for unbounded self-adjoint operators . The next level of problems - for functions of two commuting self-adjoint operators - will also be discussed .

Speaker: Franciszek Hugon Szafraniec, Jagiellonian University.

Title: A scan through dilation theory.

Abstract: I intend to share with the audience my personal view of (some) classical results of dilation theory. To make it more attractive I will wrap them up by more modern clothing of Hilbert C* modules.

Speaker: Dan Timotin, Institute of Mathematics of the Romanian Academy.

Title: Numerical ranges of contractions with finite defect numbers.

Abstract: An n-dilation of a contraction T acting on a Hilbert space H is a unitary dilation acting on $H \oplus \mathbb{C}^n$. We show that if both defect numbers of T are equal to n, then the closure of the numerical range of T is the intersection of the closures of the numerical ranges of its n-dilations. We also obtain detailed information about the geometrical properties of the numerical range of T in case $n = 1$.

Speaker: Mitsuru Uchiyama, Shimane University.

Title: Operator monotone functions, Jacobi Operators and orthogonal polynomials.

Abstract: We reveal a connection between operator monotone functions and orthogonal polynomials. Especially, we express an operator monotone function with a Jacobi operator. And then we prove the 'principal inverse' of an orthogonal polynomial is operator monotone and hence it has a holomorphic extension to the open upper half plane, namely a Pick function (or Nevanlinna function).

Speaker: Bamdad. R. Yahaghi, University of Golestan.

Title: On modules of linear transformations.

Abstract: Let D be a division ring, V and W right or left vector spaces over D, and $L(V,W)$ the $L(W)$-$L(V)$ bimodule of all right (resp. left) linear transformations from V into W. We present some basic results about submodules of $L(V,W)$.
This talk is based on joint work in progress with M. Rahimi.
